Dynamique des ondes longues et processus dispersifs, en milieu littoral et estuarien

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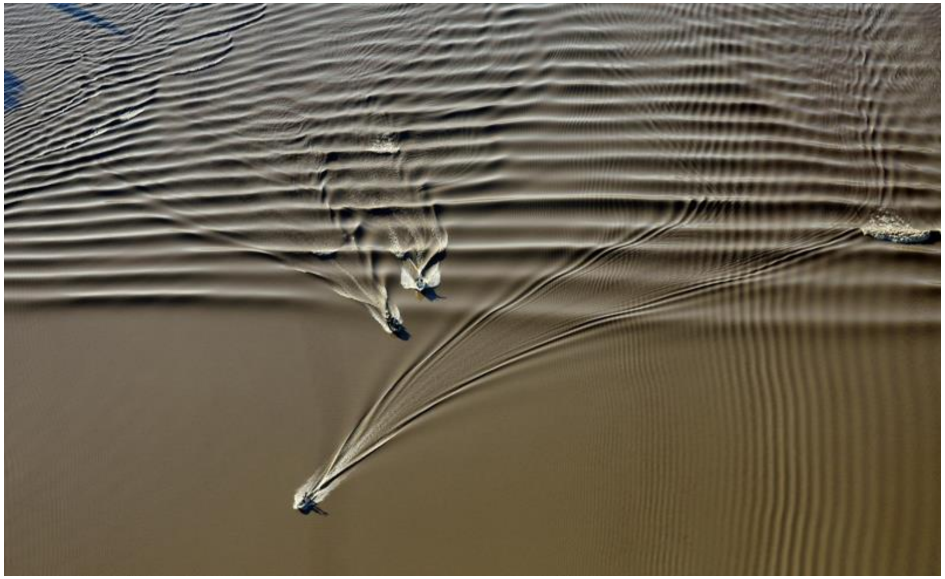
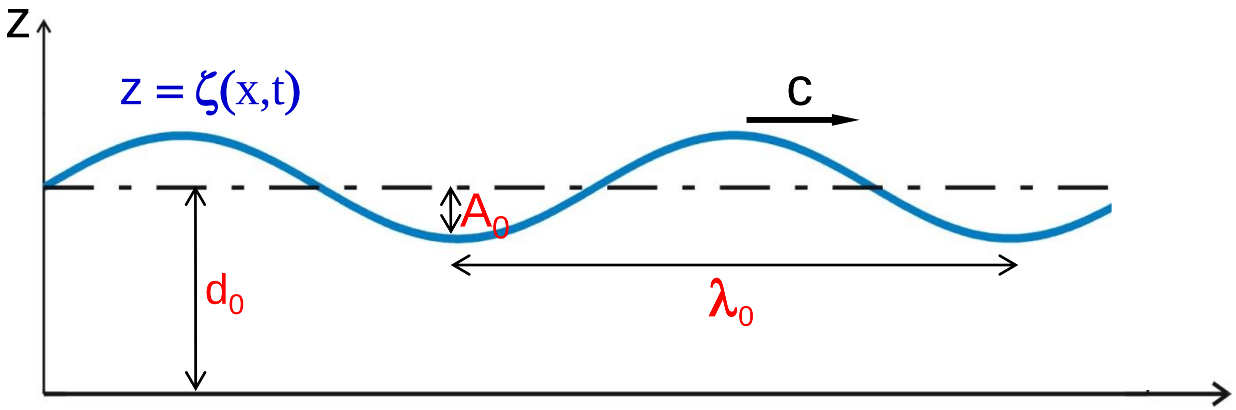
tsunami



ressaut de marée (mascaret)



d_0 , A_0 , λ_0

$$\frac{T_0}{\lambda_0/\sqrt{g d_0}} = f(A_0/d_0, d_0/\lambda_0)$$

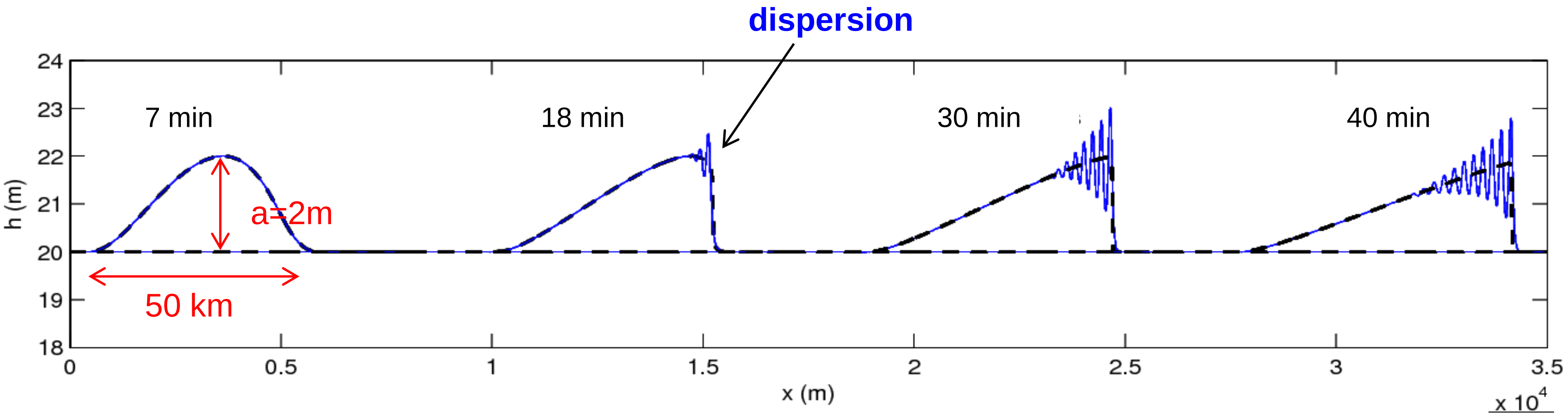
$$\varepsilon_0 = \frac{A_0}{d_0} \qquad \mu_0 = \left(\frac{d_0}{\lambda_0}\right)^2$$

ondes longues : $\mu_0 << 1$

→ tsunamis, marées, ondes infragravitaires, ondes de crue, …

ondes longues se propageant en milieu littoral et estuarien

fortes nonlinéarités → dispersion



Tissier , Bonneton et al., JCR2011

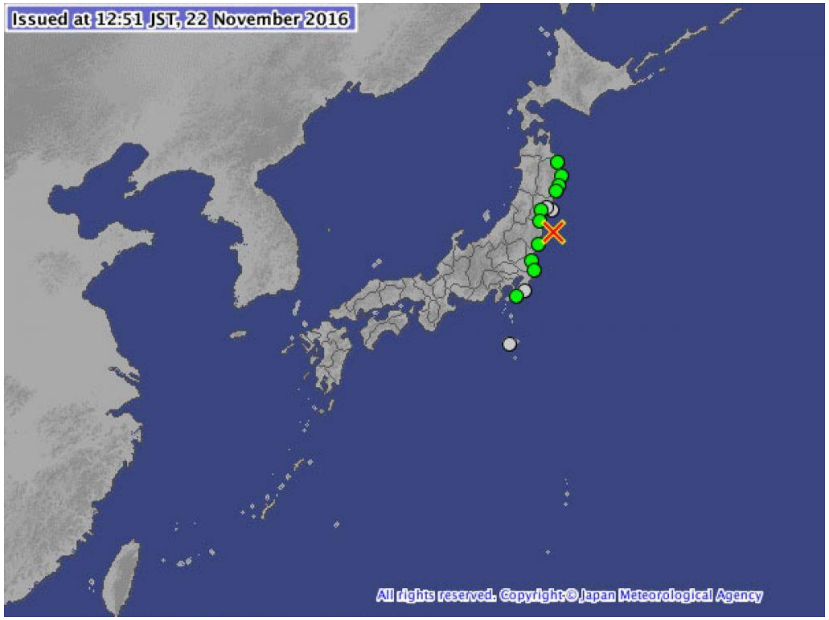
Formation et dynamique des chocs dispersifs

http://www.kohjumonline.com/anders.html

Sumatra 2004 tsunami reaching the coast of Thailand

<u>references:</u>

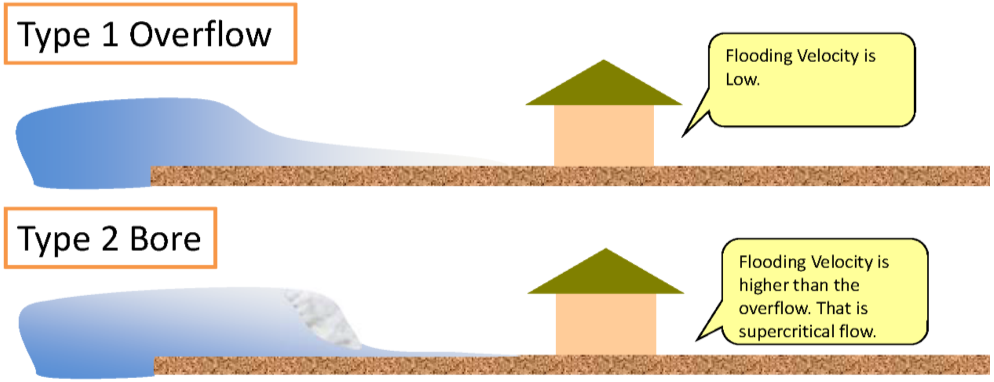
- *Grue et al. 2008*
- *Madsen et al. 2008*



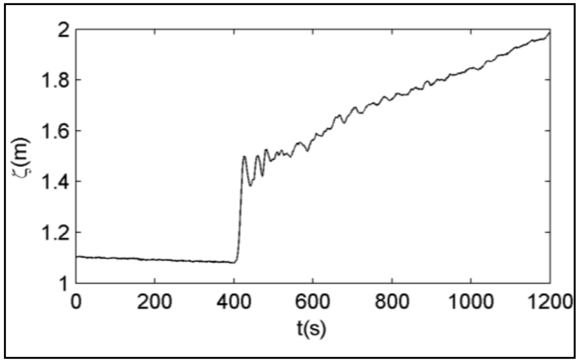
21st November 2016, Sunaoshi River in Tagajo city, Japan (earthquake 7.4)

21st November 2016, Sunaoshi River in Tagajo city, Japan (earthquake 7.4)

Impact on marine structures and buildings



Tsunamis, Arikawa et al. 2013



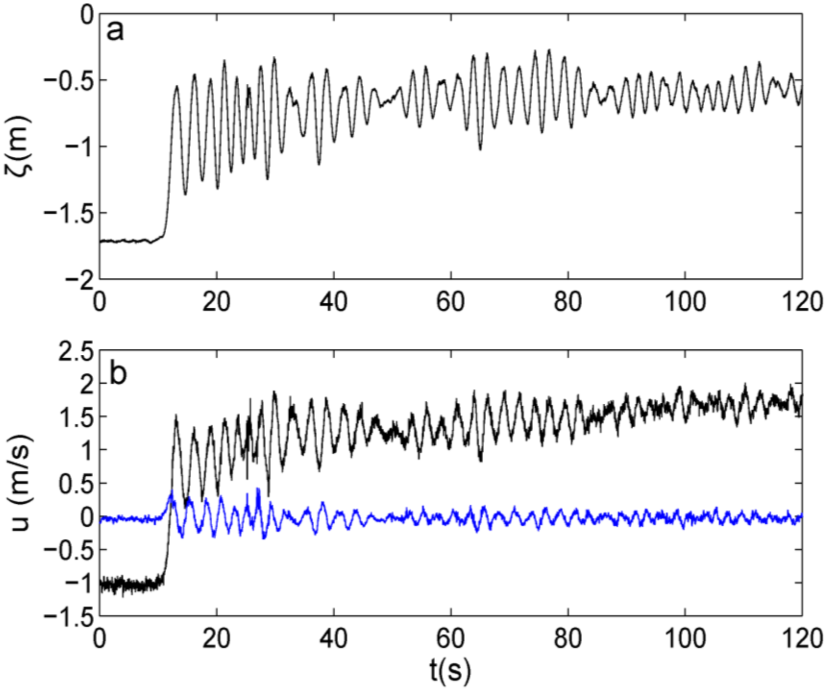
Bonneton et al. JGR 2015

Gironde estuary, Saint Pardon, Dordogne

https://vimeo.com/106090912, Jean-Marc Chauvet, Septembre 2014

Sediment transport and erosion

Tidal Bore, Bonneton et al. 2015



wind waves ($T_0 \approx 10$ s) $\Rightarrow$ **infragravity waves** ($T_0 \approx 1$ min)

Costa Rica (video: Bonneton, P. 2012)

Urumea River – San Sebastian

Dynamique des ondes longues et processus dispersifs,

en milieu littoral et estuarien

1. **Introduction**

2. **Modèles d'onde longue**

    - **notions sur les effets dispersifs**
    - **équations de Serre / Green-Naghdi**
    - **applications : simulations numériques**

3. **Distorsion des ondes longues et formation de chocs**

4. **Dynamique des ressauts de marée et mascarets**

❑ Physical Oceanography

→ *Bonneton, N., Castelle, B., J-P., Sottolichio, A. (EPOC, Bordeaux)*

→ *Frappart, F. (OMP, Toulouse)*

→ *Martins K. (Bath Univ.)*

→ *Tissier, M. (TU Delft)*

❑ Long wave modeling

→ *Lannes, D*. (IMB, Bordeaux)

→ *Ricchuito, M., Arpaia, L., Filippini, A. (INRIA, Bordeaux)*

→ *Marche, F. (IMAG, Montpellier)*

→ *Cienfuegos, R. (CIGIDEN, Chile), Barthélémy E. (LEGI, Grenoble)*

2. Modèles d'onde longue

- notions sur les effets dispersifs
- équations de Serre / Green-Naghdi
- applications : simulations numériques

champ d'onde 2D sur fond plat ($d(x) = d_0$)

$$
\begin{aligned}
\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} &= 0 \qquad z \in [-d_0, \zeta] \\
\frac{\partial w}{\partial x} - \frac{\partial u}{\partial z} &= 0 \\
\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z} &= -\frac{1}{\rho_0}\frac{\partial P}{\partial x} \\
\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + w\frac{\partial w}{\partial z} &= -g - \frac{1}{\rho_0}\frac{\partial P}{\partial z}
\end{aligned}
$$

$$
\begin{aligned}
P(z) &= P_{atm} \qquad z = \zeta \\
\frac{\partial \zeta}{\partial t} + u\frac{\partial \zeta}{\partial x} &= w \qquad z = \zeta \\
w &= 0 \qquad z = -d_0
\end{aligned}
$$

Adimensionnalisation des variables

$$x = \lambda_0 x', \quad z = d_0 z', \quad t = \frac{\lambda_0}{\sqrt{g d_0}} t',$$

$$\zeta = A_0 \zeta', \quad \Phi = \frac{A_0}{d_0} \lambda_0 \sqrt{g d_0} \Phi', \quad P = \rho_0 g d_0 P'.$$

$$
\begin{aligned}
\mu\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z} &= 0 \qquad\qquad z\in[-1,\epsilon\zeta]\\
\frac{\partial w}{\partial x}-\frac{\partial u}{\partial z} &= 0\\
\epsilon\frac{\partial u}{\partial t}+\epsilon^2 u\frac{\partial u}{\partial x}+\frac{\epsilon^2}{\mu}w\frac{\partial u}{\partial z} &= -\frac{\partial P}{\partial x}\\
\epsilon\frac{\partial w}{\partial t}+\epsilon^2 u\frac{\partial w}{\partial x}+\frac{\epsilon^2}{\mu}w\frac{\partial w}{\partial z} &= -1-\frac{\partial P}{\partial z}
\end{aligned}
$$

$$
\begin{aligned}
P(z) &= P_{atm} \qquad\qquad z=\epsilon\zeta\\
\frac{\partial \zeta}{\partial t}+\epsilon u\frac{\partial \zeta}{\partial x} &= \frac{1}{\mu}w \qquad\qquad z=\epsilon\zeta\\
w &= 0 \qquad\qquad z=-1
\end{aligned}
$$

Linéarisation des équations

Onde élémentaire :

$$\zeta(x,t) = A_0 \exp\left(2\pi i(\frac{x}{\lambda} - \frac{t}{T})\right) = A_0 \exp\left(i(kx - \omega t)\right)$$

$$\Rightarrow \quad \boxed{\omega^2 = gk \tanh(kd_0)}$$

$\zeta(x,t) = A_0 \exp\left(ik(x - \frac{\omega}{k}t)\right)$ $\Rightarrow$ vitesse de phase : $c_\phi = \frac{\omega}{k} = \frac{\lambda}{T}$

$$\boxed{c_\phi = \left(\frac{g}{k}\tanh(kd_0)\right)^{1/2}}$$

c_ϕ est une fonction décroissante de k (croissante de λ)

$$\boxed{c_\phi = \left(\frac{g}{k}\tanh(kd_0)\right)^{1/2}}$$

c_ϕ est une fonction croissante de λ

$$c_\phi = \left(\frac{g}{k}\tanh(kd_0)\right)^{1/2}$$

- $kd_0 \gg 1$: eau profonde ($\mu_0 \gg 1$)

$$c_\phi = \left(\frac{g}{k}\right)^{1/2} = \left(\frac{g\lambda}{2\pi}\right)^{1/2}$$

- $kd_0 \ll 1$: eau peu profonde ($\mu_0 \ll 1$)

$$\begin{aligned} c_\phi &= (gd_0)^{1/2}\left(1 - \frac{(kd_0)^2}{6}\right) + O((kd_0)^4) \\ c_\phi &= 1 - \frac{\mu_0 k^2}{6} + O(\mu_0^2) \end{aligned}$$

Equations faiblement dispersives : $\mu_0 \ll 1$

et entièrement nonlinéaires : $\epsilon_0 = O(1)$

intégration suivant la verticale

$$
\begin{aligned}
\int_{-1}^{\epsilon\zeta} (\mu \frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}) dz &= 0 \\
\int_{-1}^{\epsilon\zeta} (\epsilon \frac{\partial u}{\partial t} + \epsilon^2 u \frac{\partial u}{\partial x} + \frac{\epsilon^2}{\mu} w \frac{\partial u}{\partial z} + \frac{\partial P}{\partial x}) dz &= 0
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial \zeta}{\partial t} + \frac{\partial hU}{\partial x} &= 0 \\
\epsilon \frac{\partial U}{\partial t} + \epsilon^2 U \frac{\partial U}{\partial x} + \frac{\epsilon^2}{h} \frac{\partial}{\partial x} \left(\int_{-1}^{\epsilon\zeta} (u^2 - U^2)\ dz \right) &= -\frac{1}{h} \frac{\partial}{\partial x} \left(\int_{-1}^{\epsilon\zeta} P\ dz \right)
\end{aligned}
$$

où $U = \frac{1}{h} \left(\int_{-1}^{\epsilon\zeta} u\ dz \right)$

$$\epsilon \frac{\partial w}{\partial t} + \epsilon^2 u \frac{\partial w}{\partial x} + \frac{\epsilon^2}{\mu} w \frac{\partial w}{\partial z} = -1 - \frac{\partial P}{\partial z}$$

$$1 + \epsilon \Gamma = -\frac{\partial P}{\partial z}$$

$\Gamma = \frac{\partial w}{\partial t} + \epsilon u \frac{\partial w}{\partial x} + \frac{\epsilon}{\mu} w \frac{\partial w}{\partial z}$ est l'accélération verticale des particules fluides

$$P = \epsilon\zeta - z + \epsilon \int_z^{\epsilon\zeta} \Gamma \; d\xi$$

$$\int_{-1}^{\epsilon\zeta} P \; dz = \frac{1}{2} h^2 + \epsilon \int_{-1}^{\epsilon\zeta} \; dz \int_z^{\epsilon\zeta} \Gamma \; d\xi$$

$$\int_{-1}^{\epsilon\zeta} P \; dz = \frac{1}{2} h^2 + \epsilon \int_{-1}^{\epsilon\zeta} \Gamma(z+1) \; dz \; .$$

$$
\begin{aligned}
\frac{\partial \zeta}{\partial t} + \frac{\partial hU}{\partial x} &= 0 \\
\frac{\partial U}{\partial t} + \epsilon U \frac{\partial U}{\partial x} + \frac{\partial \zeta}{\partial x} + \frac{\epsilon}{h}\frac{\partial}{\partial x}\left(\int_{-1}^{\epsilon\zeta} (u^2 - U^2)\ dz \right) &= -\frac{1}{h}\frac{\partial}{\partial x}\left(\int_{-1}^{\epsilon\zeta} (z+1)\Gamma\ dz \right)
\end{aligned}
$$

méthode de perturbation de Φ en série des puissances de μ :

$$\Phi = \sum_{j=0}^{N} \mu^j \Phi_j \ .$$

$$\mu \frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial z^2} = 0$$

$$\frac{\partial^2 \Phi_0}{\partial z^2} + \mu \left(\frac{\partial^2 \Phi_0}{\partial x^2} + \frac{\partial^2 \Phi_1}{\partial z^2} \right) + ... = 0 \ .$$

- On en déduit que $\frac{\partial^2 \Phi_0}{\partial z^2} = 0$ et donc $\frac{\partial \Phi_0}{\partial z} = c1(x,t)$.
  D'après la condition au limite au fond on obtient $\frac{\partial \Phi_0}{\partial z} = 0$ et donc :

$$\Phi_0 = \Phi(z = \epsilon\zeta) = \psi(x,t) \ .$$

- A un ordre supérieur en μ on trouve :

$$\Phi_1 = -\frac{1}{2}((z+1)^2 - h^2)\frac{\partial^2 \psi}{\partial x^2}$$

$$\Phi_0 = \psi(x,t) \qquad \Phi_1 = -\frac{1}{2}((z+1)^2 - h^2)\frac{\partial^2 \psi}{\partial x^2}$$

$$u = \frac{\partial \Phi}{\partial x} = \frac{\partial \psi}{\partial x} + \mu\epsilon h \frac{\partial \zeta}{\partial x}\frac{\partial^2 \psi}{\partial x^2} - \frac{\mu}{2}((z+1)^2 - h^2)\frac{\partial^3 \psi}{\partial x^3} + O(\mu^2)$$

$$U = \frac{\partial \psi}{\partial x} + \mu\epsilon h \frac{\partial \zeta}{\partial x}\frac{\partial^2 \psi}{\partial x^2} + \frac{\mu}{3}h^2\frac{\partial^3 \psi}{\partial x^3} + O(\mu^2)$$

$$\begin{aligned} u &= U - \frac{\mu}{2}(z+1)^2\frac{\partial^2 U}{\partial x^2} + \frac{\mu}{6}h^2\frac{\partial^2 U}{\partial x^2} + O(\mu^2) \\ w &= \frac{\partial \Phi}{\partial z} = -\mu(z+1)\frac{\partial U}{\partial x} + O(\mu^2) \end{aligned}$$

$$
\begin{aligned}
u &= U - \frac{\mu}{2}(z+1)^2 \frac{\partial^2 U}{\partial x^2} + \frac{\mu}{6} h^2 \frac{\partial^2 U}{\partial x^2} + O(\mu^2) \\
w &= \frac{\partial \Phi}{\partial z} = -\mu (z+1) \frac{\partial U}{\partial x} + O(\mu^2)
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial \zeta}{\partial t} + \frac{\partial h U}{\partial x} &= 0 \\
\frac{\partial U}{\partial t} + \epsilon U \frac{\partial U}{\partial x} + \frac{\partial \zeta}{\partial x} + \frac{\epsilon}{h} \frac{\partial}{\partial x} \left(\int_{-1}^{\epsilon\zeta} (u^2 - U^2)\, dz \right) &= -\frac{1}{h} \frac{\partial}{\partial x} \left(\int_{-1}^{\epsilon\zeta} (z+1)\Gamma\, dz \right)
\end{aligned}
$$

$$
\int_{-1}^{\epsilon\zeta} (u^2 - U^2)\, dz = O(\mu^2)
$$

$$
\Gamma = -\mu(z+1) \left(\frac{\partial^2 U}{\partial x \partial t} + \epsilon U \frac{\partial^2 U}{\partial x^2} - \epsilon \left(\frac{\partial U}{\partial x} \right)^2 \right)
$$

$$
\begin{aligned}
\frac{\partial \zeta}{\partial t} + \frac{\partial hU}{\partial x} &= 0 \\
\frac{\partial U}{\partial t} + \epsilon U \frac{\partial U}{\partial x} + \frac{\partial \zeta}{\partial x} &= \frac{\mu}{3h} \frac{\partial}{\partial x} \left(h^3 \left(\frac{\partial^2 U}{\partial x \partial t} + \epsilon U \frac{\partial^2 U}{\partial x^2} - \epsilon \left(\frac{\partial U}{\partial x} \right)^2 \right) \right)
\end{aligned}
$$

- $O(\mu)$: équations de Saint Venant (entièrement nonlinéaires et non dispersives)
- $O(\mu^2)$ et $\epsilon = O(\mu)$: équations de Boussinesq (faiblement non linéaires et dispersives)
- $O(\mu^2)$ et $\epsilon = O(1)$: équations de Serre / Green Naghdi (entièrement non linéaires et faiblement dispersives)

Relation de dispersion

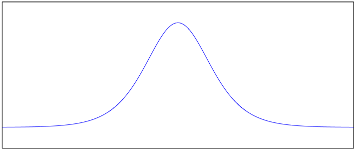
$$
\begin{aligned}
\frac{\partial \zeta}{\partial t} + \frac{\partial U}{\partial x} &= 0 \\
\frac{\partial U}{\partial t} + \frac{\partial \zeta}{\partial x} &= \frac{\mu}{3} \frac{\partial^3 U}{\partial^2 x \partial t}
\end{aligned}
$$

$$
c_\phi = \left(1 + \frac{\mu k^2}{3}\right)^{-1/2} \simeq 1 - \frac{\mu k^2}{6}
$$

Onde solitaire

$$h(x,t) = d_0 + A_0 \operatorname{sech}^2\left(K(x - Ct)\right)$$

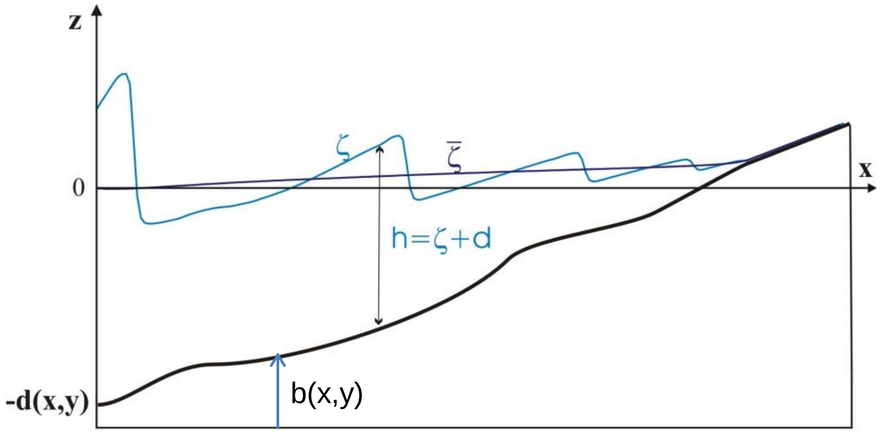
$$\begin{aligned} K &= \frac{1}{d_0}\left(\frac{3\epsilon_0}{4(1+\epsilon_0)}\right)^{1/2} \\ C &= (gd_0)^{1/2}(1+\epsilon_0)^{1/2} \\ U &= C(1 - \frac{d_0}{h}) \end{aligned}$$



$$\partial_t \zeta + \nabla\cdot(h\mathbf{u}) = 0$$

$$\partial_t \mathbf{u} + \varepsilon(\mathbf{u}\cdot\nabla)\mathbf{u} + \nabla\zeta = \mu\mathcal{D} + O(\mu^2)$$

Lannes and Bonneton (2009)



$$\mathcal{D} = -\mathcal{T}[h,b]\mathbf{u}_t - \varepsilon\mathcal{Q}[h,b](\mathbf{u})$$

where the linear operator $\mathcal{T}[h,b]$ is defined as

$$\mathcal{T}[h,b]W = -\frac{1}{3h}\nabla(h^3\nabla\cdot W) + \frac{1}{2h}[\nabla(h^2\nabla b\cdot W) - h^2\nabla b\nabla\cdot W] + \nabla b\nabla b\cdot W$$

and the quadratic term $\mathcal{Q}[h,b](\mathbf{u})$ is given by

$$\begin{aligned}\mathcal{Q}[h,b](\mathbf{u}) &= -\frac{1}{3h}\nabla\left(h^3((\mathbf{u}\cdot\nabla)(\nabla\cdot\mathbf{u}) - (\nabla\cdot\mathbf{u})^2)\right)\\ &\quad + \frac{1}{2h}[\nabla(h^2(\mathbf{u}\cdot\nabla)^2 b) - h^2((\mathbf{u}\cdot\nabla)(\nabla\cdot\mathbf{u}) - (\nabla\cdot\mathbf{u})^2)\nabla b] + ((\mathbf{u}\cdot\nabla)^2 b)\nabla b\end{aligned}$$

Reformulation of SGN equations

$$\begin{aligned} \partial_t h + \nabla \cdot (h\mathbf{u}) &= 0 \\ \partial_t (h\mathbf{u}) + \nabla \cdot (h\mathbf{u} \otimes \mathbf{u}) + \nabla(\frac{1}{2} g h^2) &= -gh\nabla b \\ &+ \frac{1}{\alpha} gh\nabla\zeta - (I + \alpha h \mathcal{T} \frac{1}{h})^{-1} [\frac{1}{\alpha} gh\nabla\zeta + h\mathcal{Q}_1(\mathbf{u})] \end{aligned}$$

$\mathcal{Q}_1(\mathbf{u}) = \mathcal{Q}(\mathbf{u}) - \mathcal{T}((\mathbf{u} \cdot \nabla)\mathbf{u})$ only involves second order derivatives of $\mathbf{u}$

α −− > improved dispersive properties (Madsen et al., 1991)

$kd_0 \leq 3$

Bonneton, Chazel, Lannes, Marche and Tissier (2011)

$$\partial_t h + \nabla \cdot (h\mathbf{u}) = 0$$

$$\partial_t (h\mathbf{u}) + \nabla \cdot (h\mathbf{u} \otimes \mathbf{u}) + \nabla(\frac{1}{2} g h^2) = -gh\nabla b + \frac{1}{\alpha} gh\nabla\zeta - (I + \alpha h \mathcal{T} \frac{1}{h})^{-1} [\frac{1}{\alpha} gh\nabla\zeta + h\mathcal{Q}_1(\mathbf{u})]$$

Lannes and Marche (2014) have proposed a new formulation where

the **operator to invert** is **time independent**

$\rightarrow$ **a considerable decrease of the computational time!**

$$\partial_t h + \nabla \cdot (h\mathbf{u}) = 0$$

$$\partial_t (h\mathbf{u}) + \nabla \cdot (h\mathbf{u} \otimes \mathbf{u}) + \nabla(\frac{1}{2} g h^2) = -gh\nabla b$$

$$+\frac{1}{\alpha} gh\nabla\zeta - (I + \alpha h \mathcal{T} \frac{1}{h})^{-1} [\frac{1}{\alpha} gh\nabla\zeta + h\mathcal{Q}_1(\mathbf{u})]$$

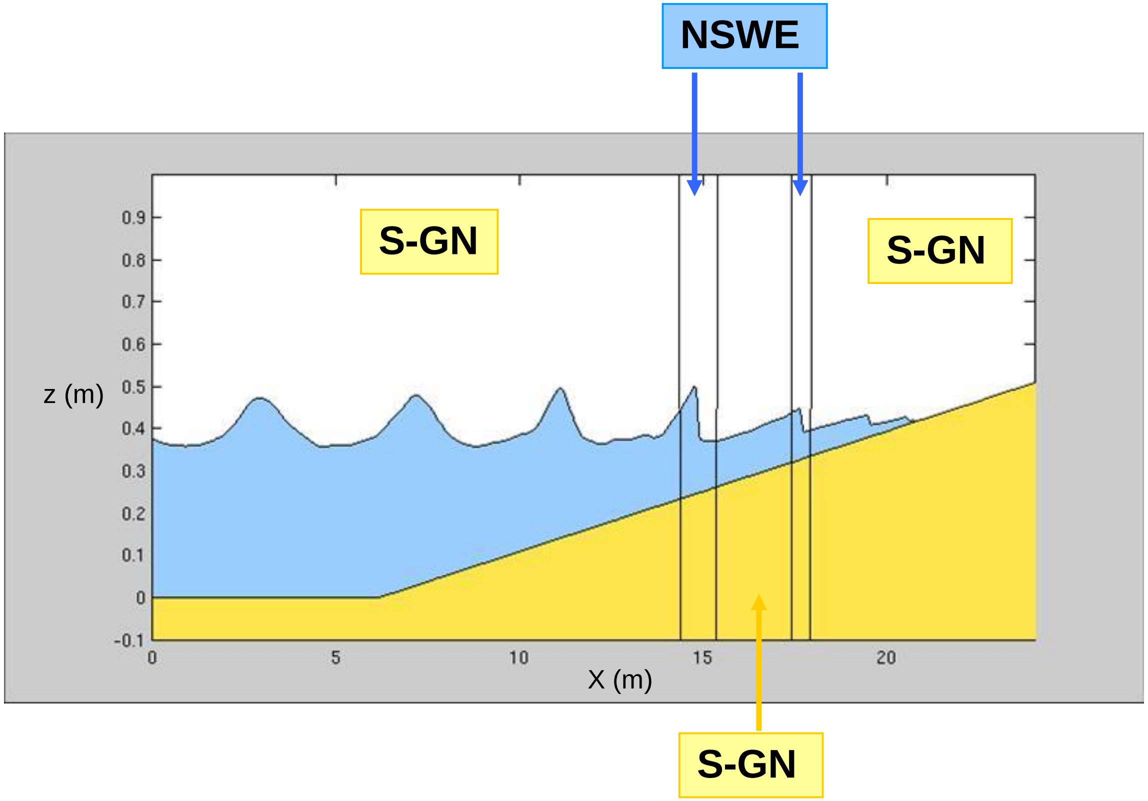
- **Numerical strategy: decoupling between the hyperbolic and the elliptic parts**

  e.g. Duran and Marche (2016), Filippini et al. (2017)

- **Strategy for wave breaking: description of broken-wave fronts as shocks by the NSWE, by skipping the dispersive step S2**

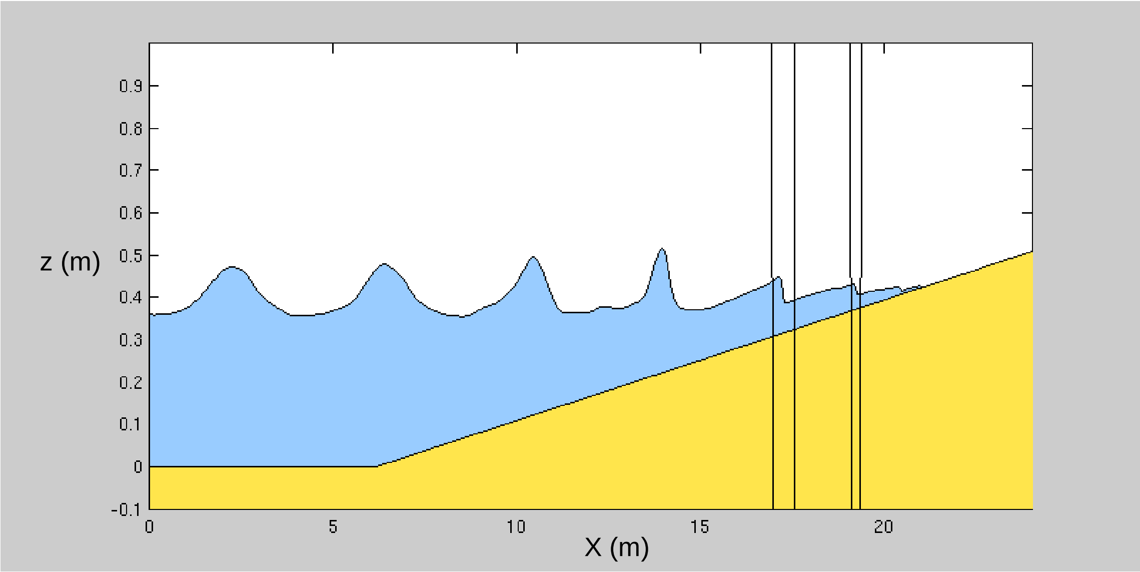
  Bonneton et al. (2011)

Shoaling and breaking of regular waves over a sloping beach



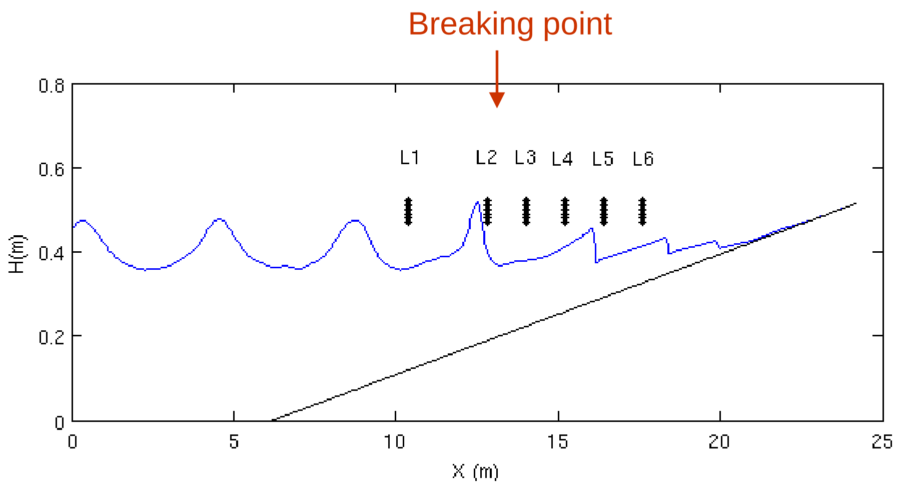
Tissier et al. (2012)

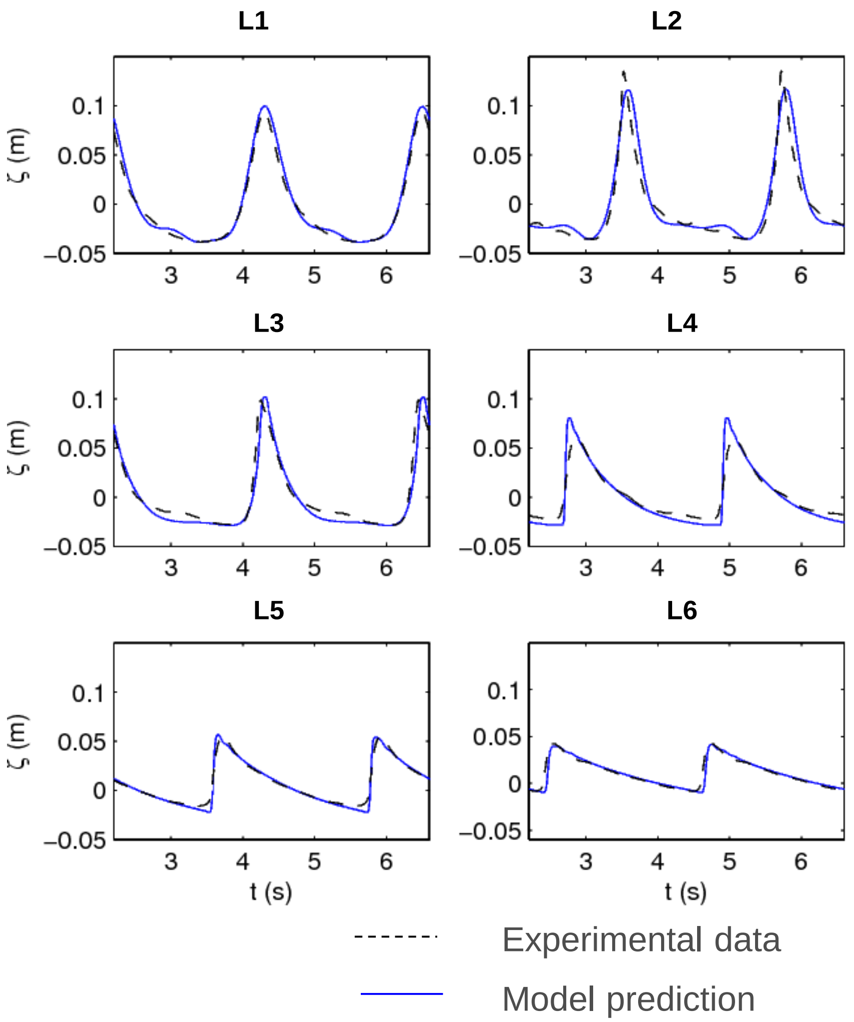
Shoaling and breaking of regular waves over a sloping beach



Shoaling and breaking of regular waves over a sloping beach

Validation with Cox (1995) experiments

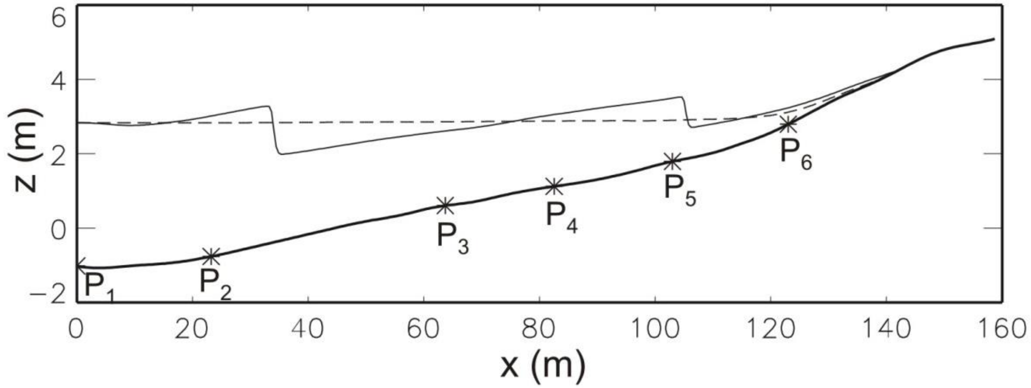




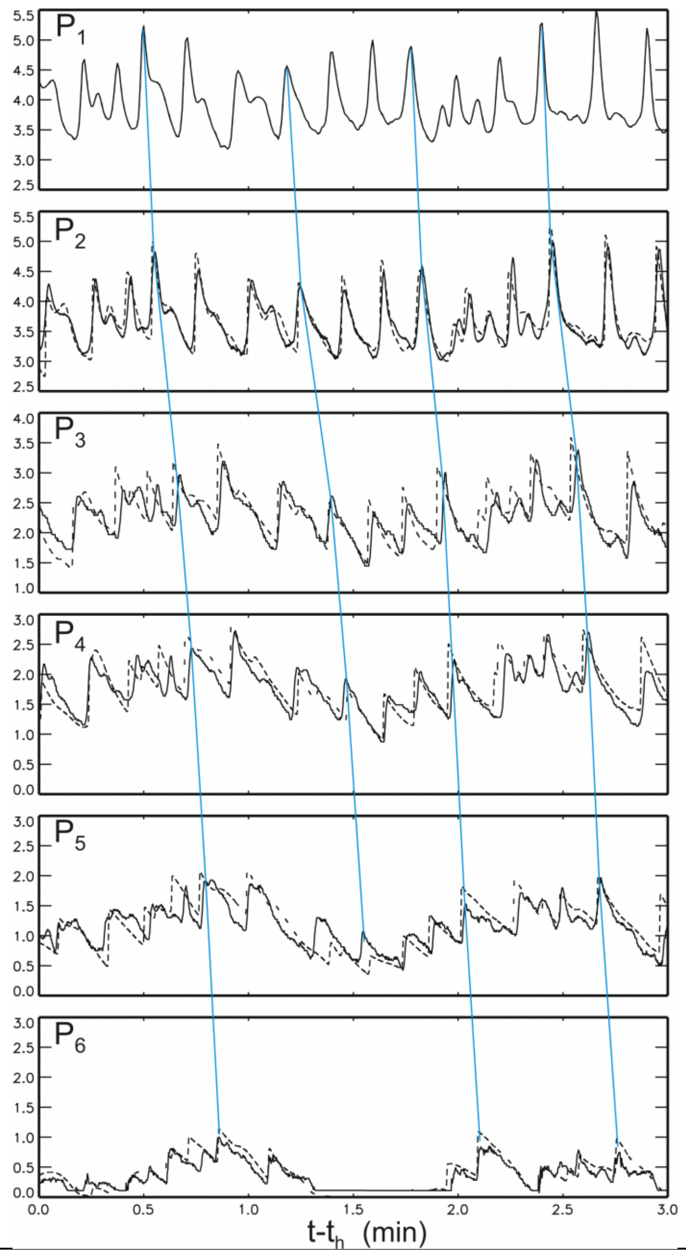
Comparison with field data

Truc Vert Beach 2001

- Offshore wave conditions: $\theta \approx 0°$, Hs=3 m, Ts=12 s
- Maximum surf zone width: 500 m

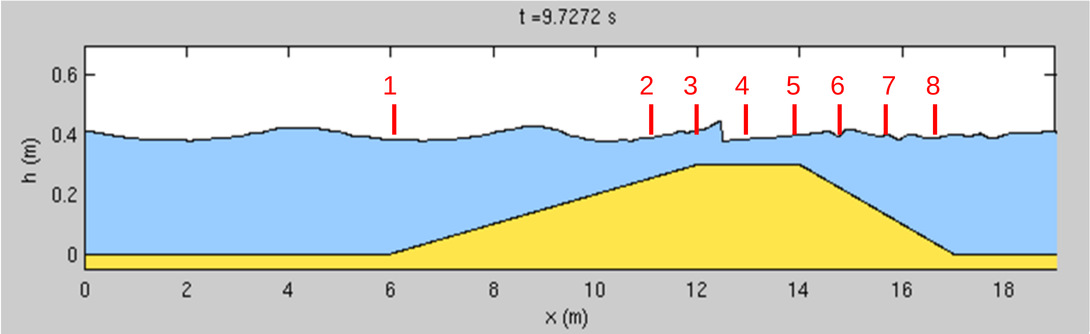


Bottom topography and pressure sensor locations



Periodic waves breaking over a bar

Validation with Beji and Battjes (1993) experiments



Tissier et al. (2012)

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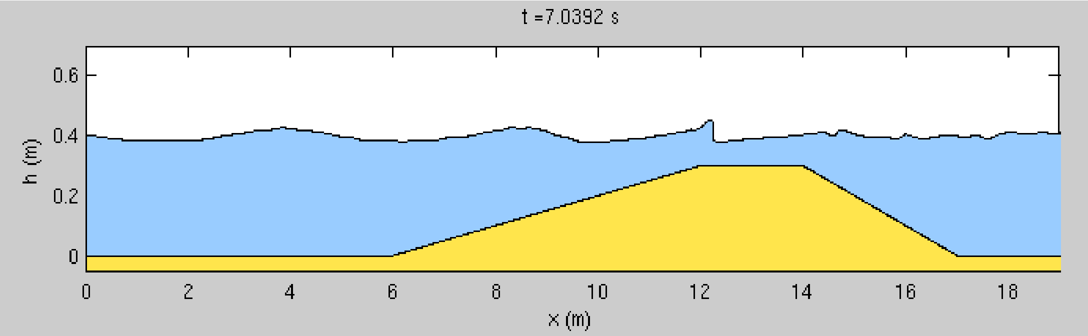

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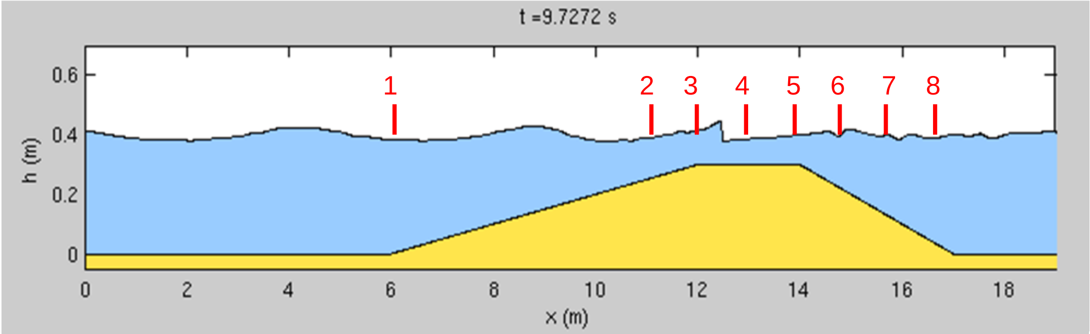
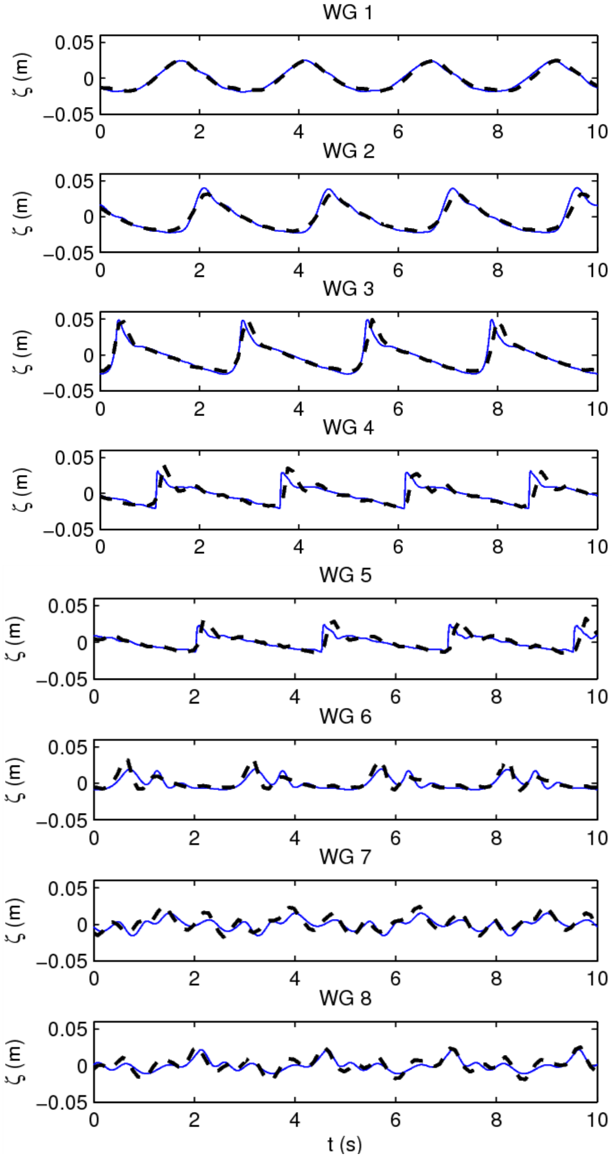




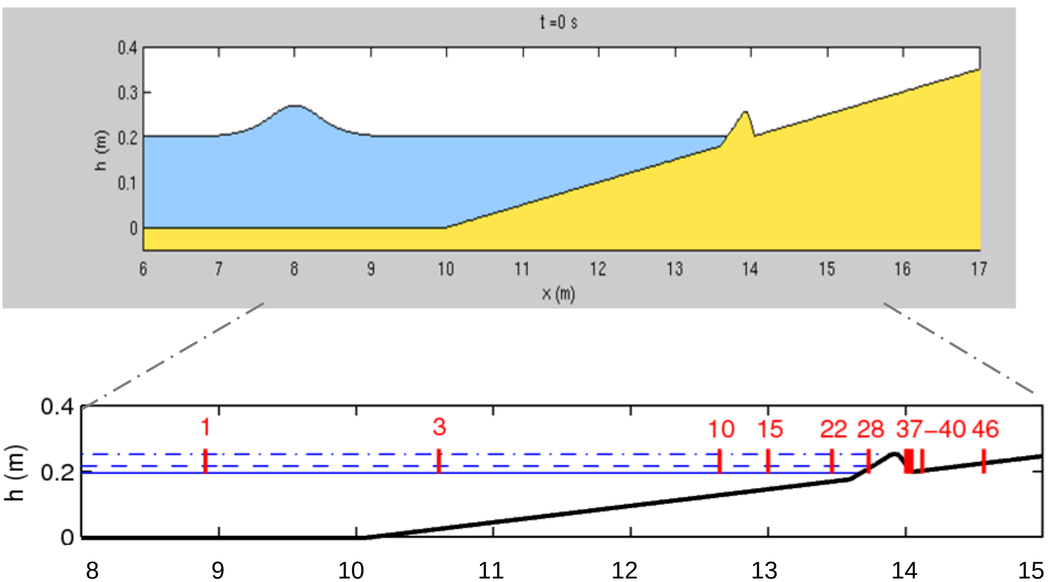
Tissier et al. (2012)

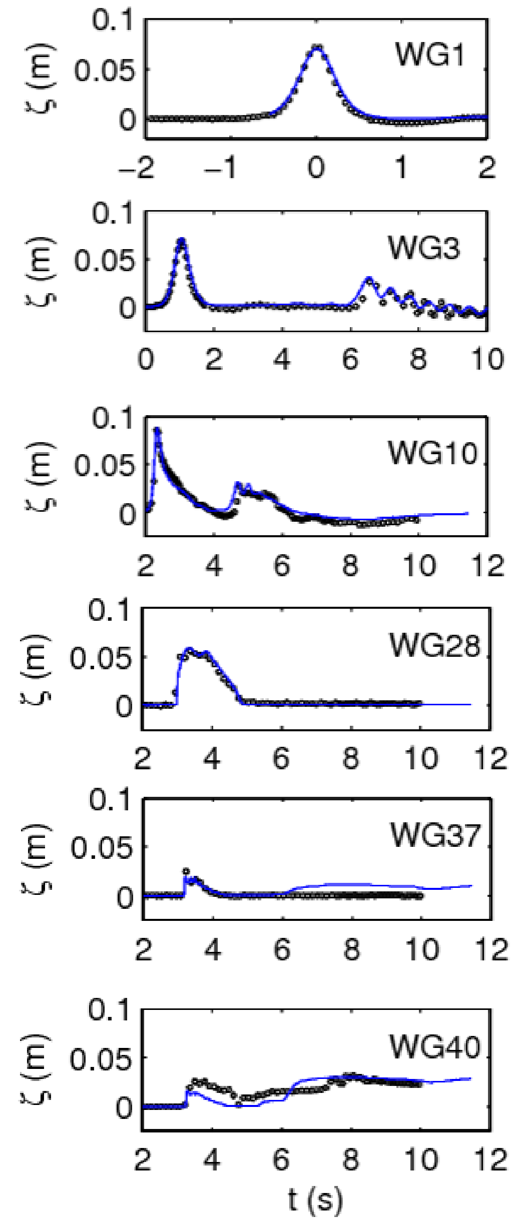
Laboratory data

Model prediction

Wave overtopping and multiple shorelines

Solitary waves overtopping a seawall (Hsiao and Lin, 2010)



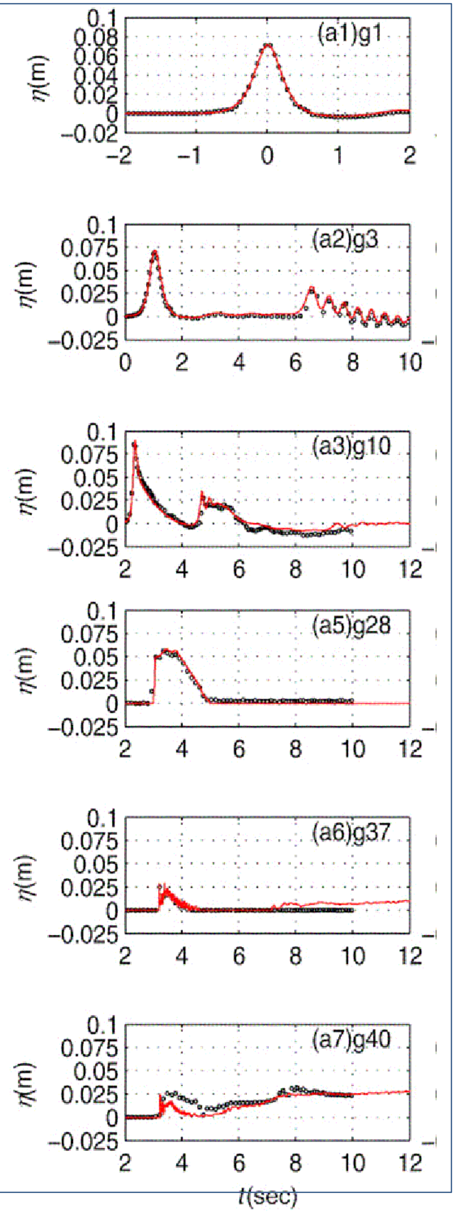


Wave overtopping and multiple shorelines

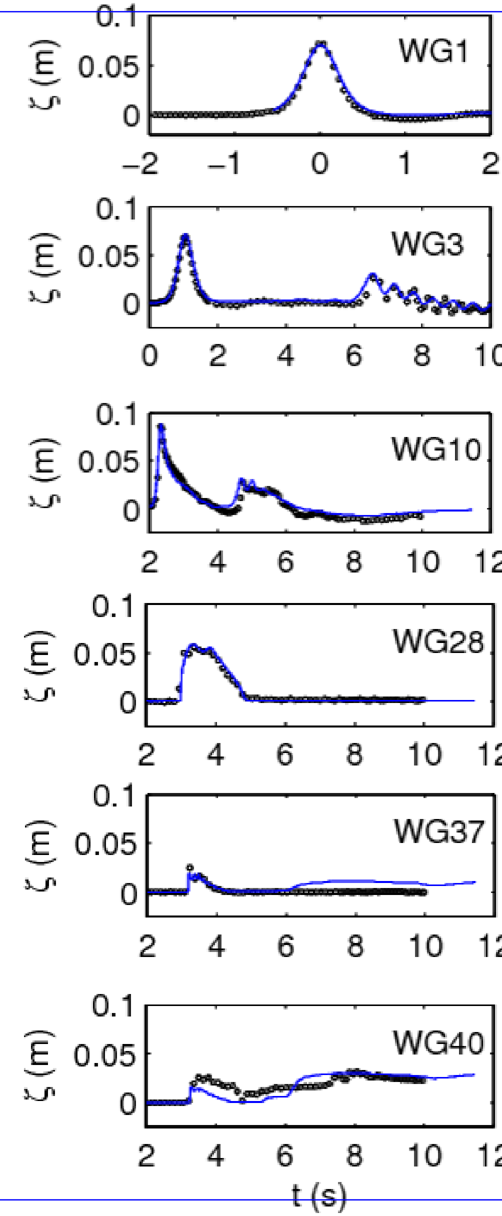
Hsiao et Lin (2010)
COBRAS model

2D VOF model

RANS equations K-ε

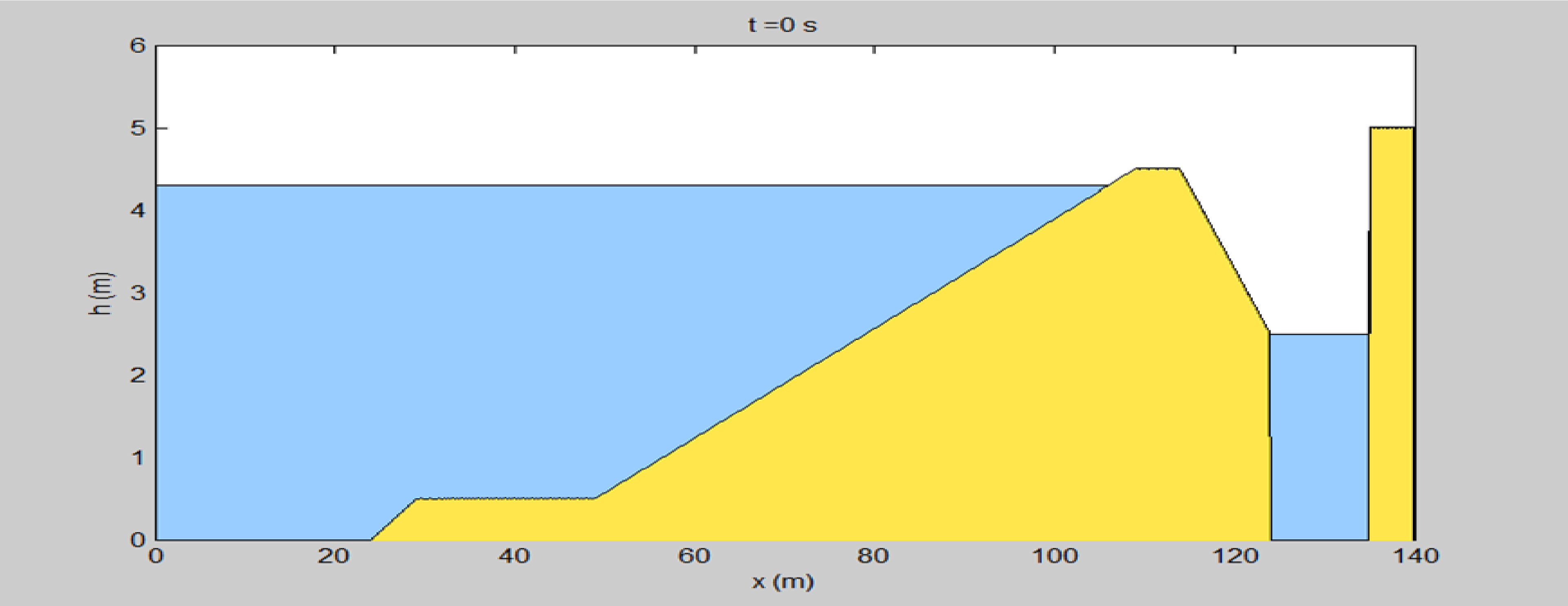


SURF-GN



Wave overtopping and multiple shorelines

BARDEX II (HYDRALAB project, Delta Flumes, PI: Gerd Masselink)
Barrier Dynamics Experiment : shallow water sediment transport processes in the inner surf, swash and overwash zone.



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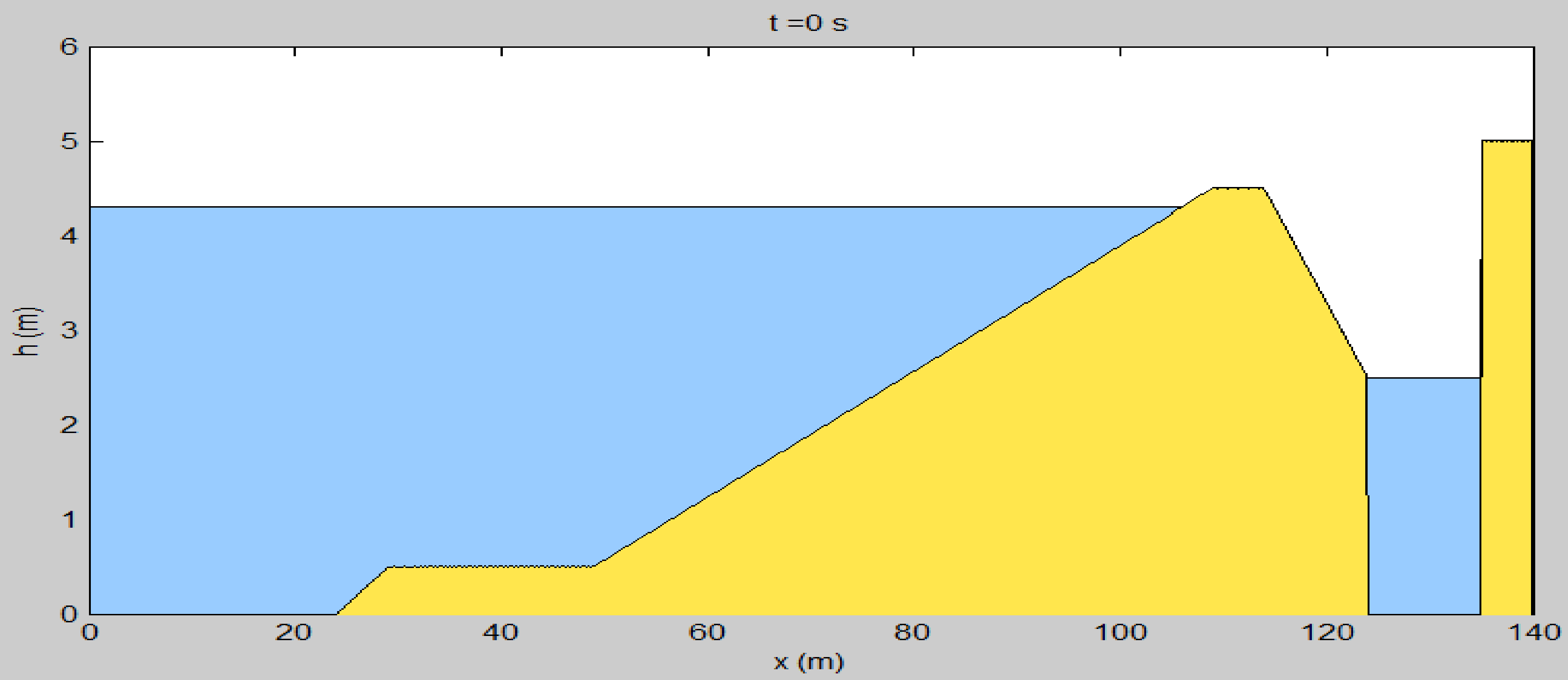
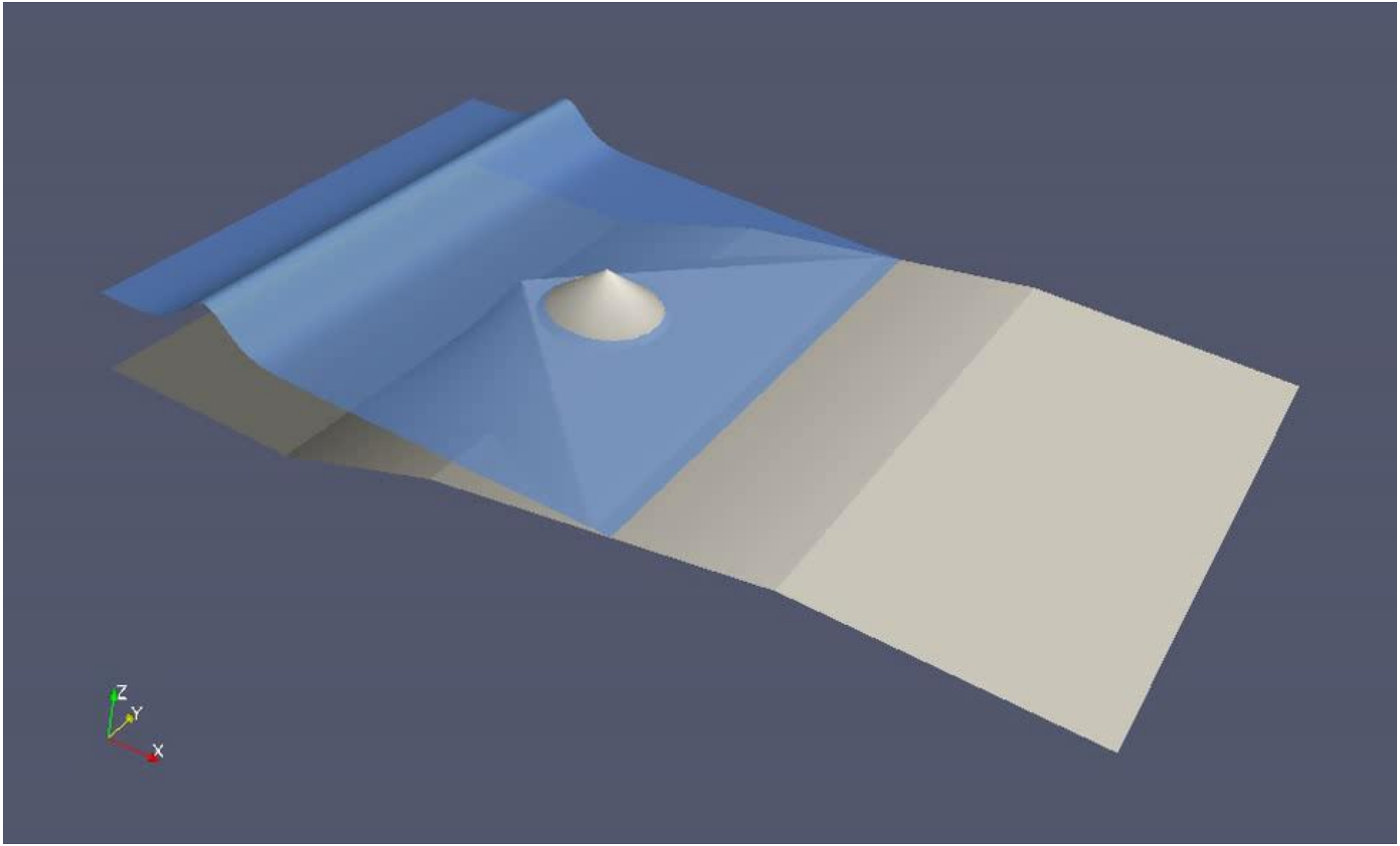
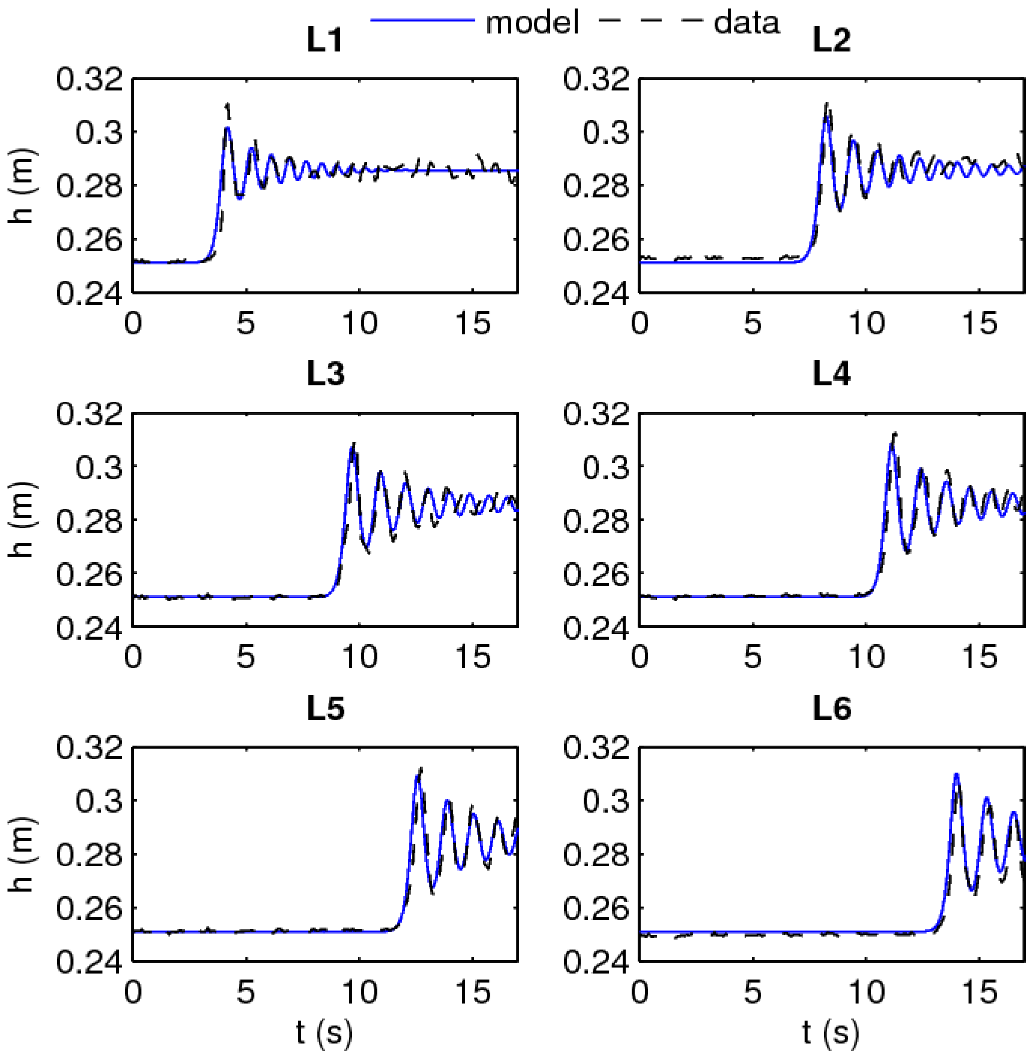




Marche et Lannes, 2014

Undular bore (dispersive choc)



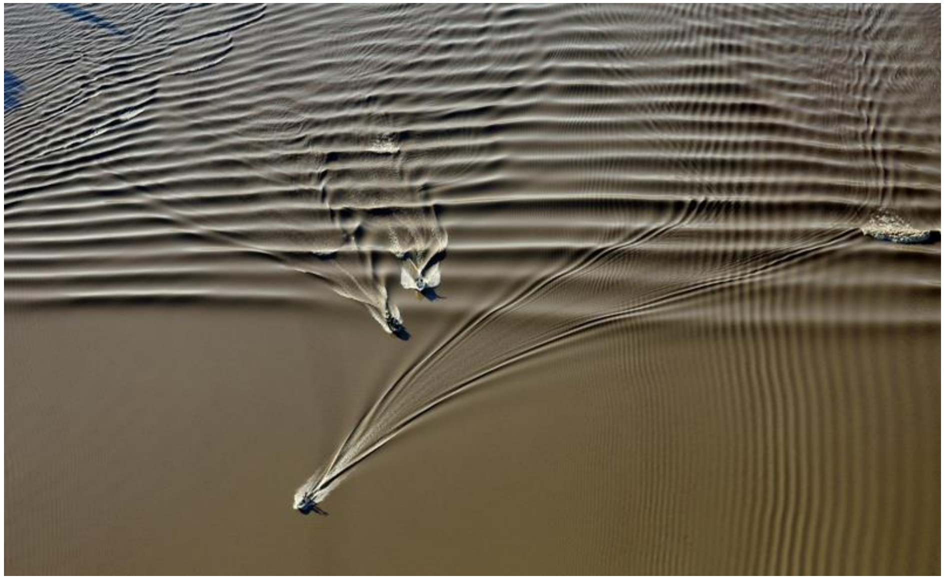
Data from Soares-Frazao et Zech (2002), **Fr = 1.104**

3. Distorsion des ondes longues et formation de chocs

What are the conditions for tsunami-like bore formation

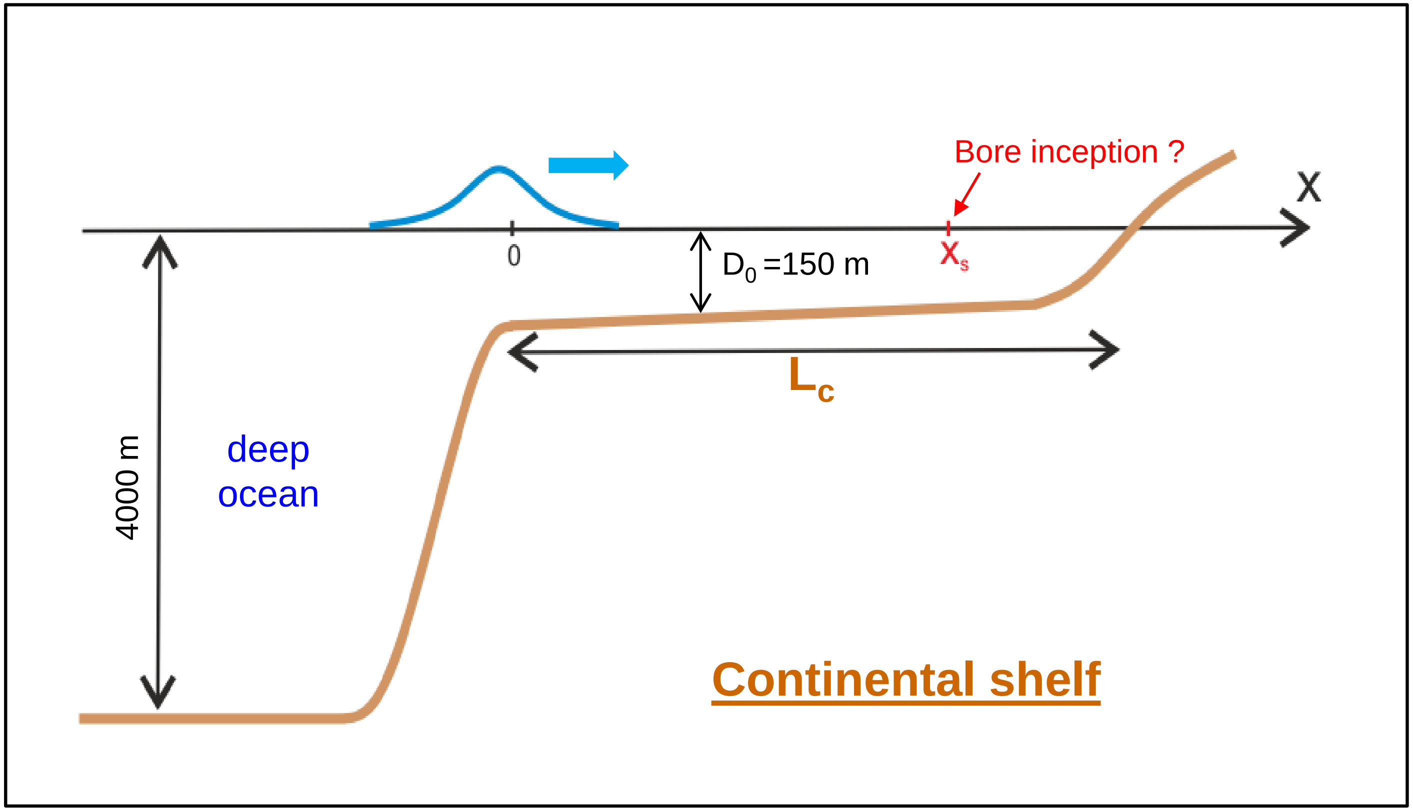
in coastal and estuarine environments?

tsunami bore



tidal bore

Basic conditions for bore formation



- the continental shelf is relatively flat

- D_0 , A_0 , $\omega_0 = 2\pi / T_0$

$$L_{w0} = \frac{\sqrt{g D_0}}{\omega_0}$$

$$\epsilon_0 = \frac{A_0}{D_0} \qquad \mu_0 = \frac{D_0^2}{L_{w0}^2}$$

Basic conditions for bore formation

$\mu_0 << 1 \quad \varepsilon_0 = O(1)$

$$\frac{\partial \zeta}{\partial t} + \vec{\nabla}.((1+\epsilon_0 \zeta)\vec{u}) = 0$$
$$\frac{\partial \vec{u}}{\partial t} + \epsilon_0(\vec{u}.\vec{\nabla})\vec{u} + \vec{\nabla}\zeta = \mu_0 D$$

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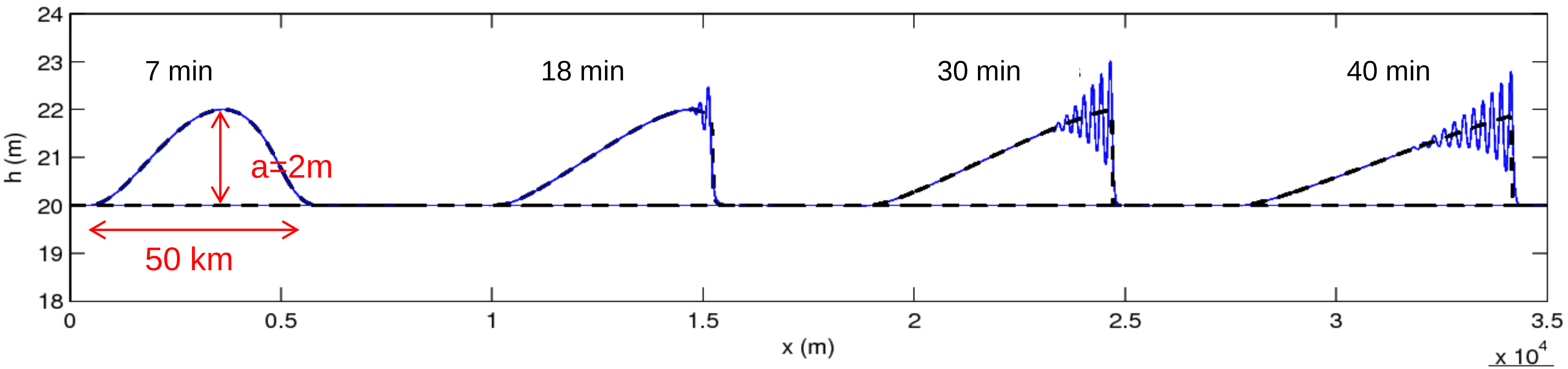

Serre Green Naghdi model

Tissier , Bonneton et al., JCR2011

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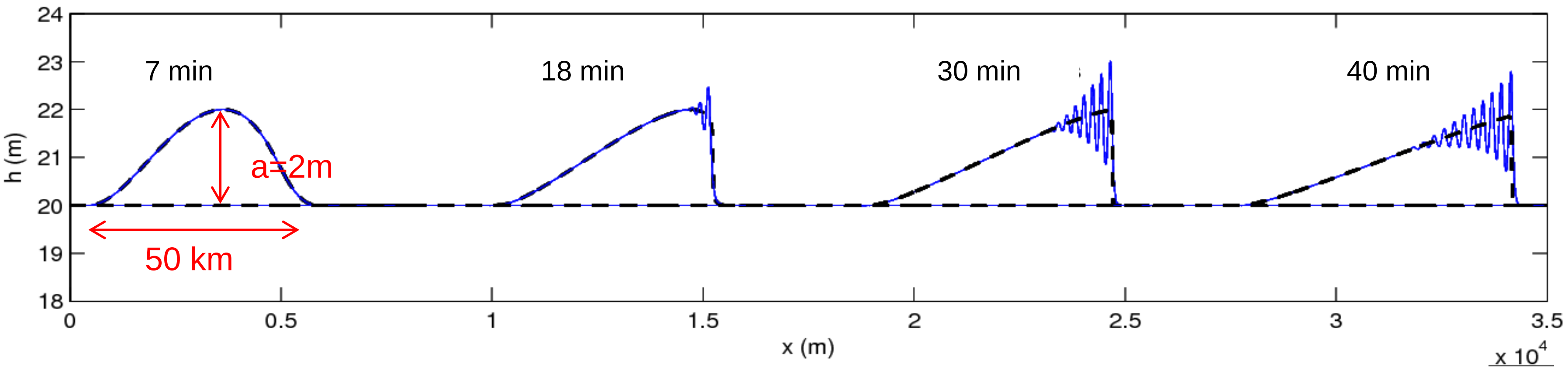


Serre Green Naghdi model

Tissier , Bonneton et al., JCR2011

Basic conditions for bore formation

$$\begin{aligned}\frac{\partial}{\partial t}(u-2c)+(u-c)\frac{\partial}{\partial x}(u-2c) &= 0\\ \frac{\partial}{\partial t}(u+2c)+(u+c)\frac{\partial}{\partial x}(u+2c) &= 0\end{aligned}$$

$$c=(gh)^{1/2} \qquad c_0=(gD_0)^{1/2}$$

$$\text{One way} \quad \Rightarrow \quad u-2c=-2c_0$$

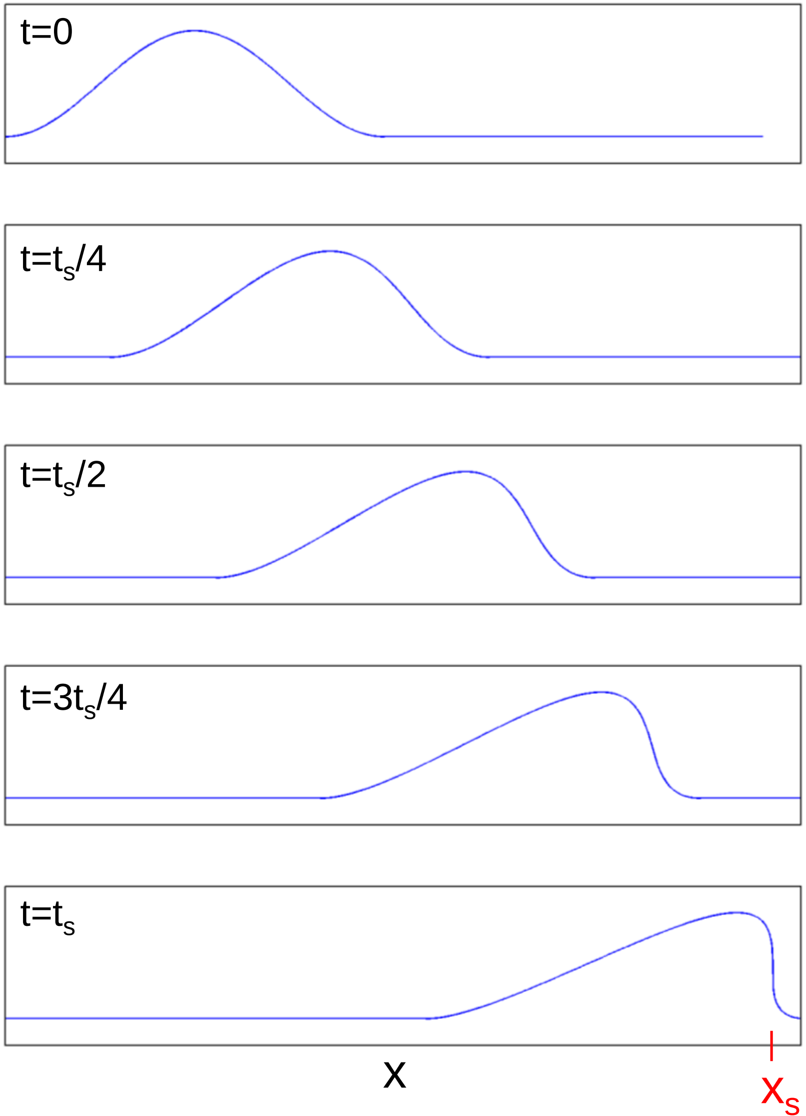
$$\frac{\partial h}{\partial t}+C_h(h)\frac{\partial h}{\partial x}=0 \qquad\qquad C_h=3c-2c_0$$

$$\left\{\begin{aligned} h(x,t) &= h(x_0,t=0) \qquad \text{along}\\ \frac{dx}{dt} &= C_h \qquad \text{or} \qquad x=x_0+C_h(x_0,t=0)t\end{aligned}\right.$$

$$h(x,t)=h_0(x-C_h(x_0)t)$$

Basic conditions for bore formation

$$h(x,t) = h_0(x - C_h(x_0)t)$$

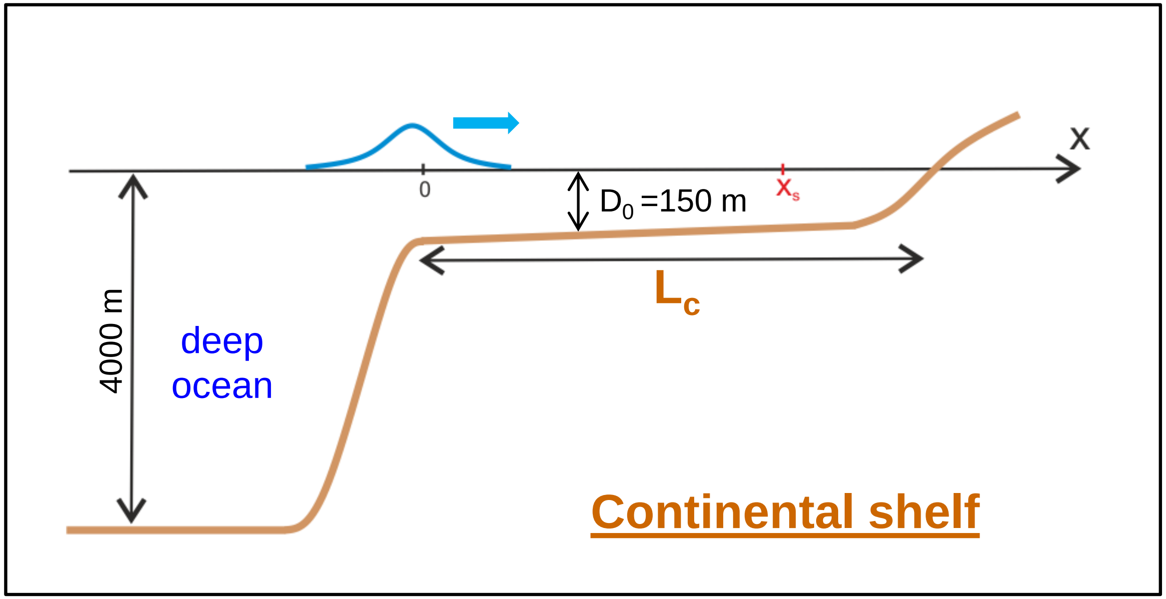


$$\frac{\partial h}{\partial x} = \frac{\frac{\partial h_0}{\partial x_0}}{1 + \frac{\partial C_h}{\partial x_0} t} = \frac{\frac{\partial h_0}{\partial x_0}}{1 + \frac{3c_0}{2\sqrt{D_0 h_0}} \frac{\partial h_0}{\partial x_0} t}$$

$$t_s = -\frac{2}{3} \frac{D_0}{c_0 \frac{\partial h_0}{\partial x_0})_s} \quad \Rightarrow \quad x_s = -\frac{2}{3} \frac{D_0 C_{h_s}}{c_0 \frac{\partial h_0}{\partial x_0})_s}$$

$$x_s \simeq -\frac{2}{3} \frac{D_0}{\frac{\partial h_0}{\partial x_0})_s}$$

Basic conditions for bore formation

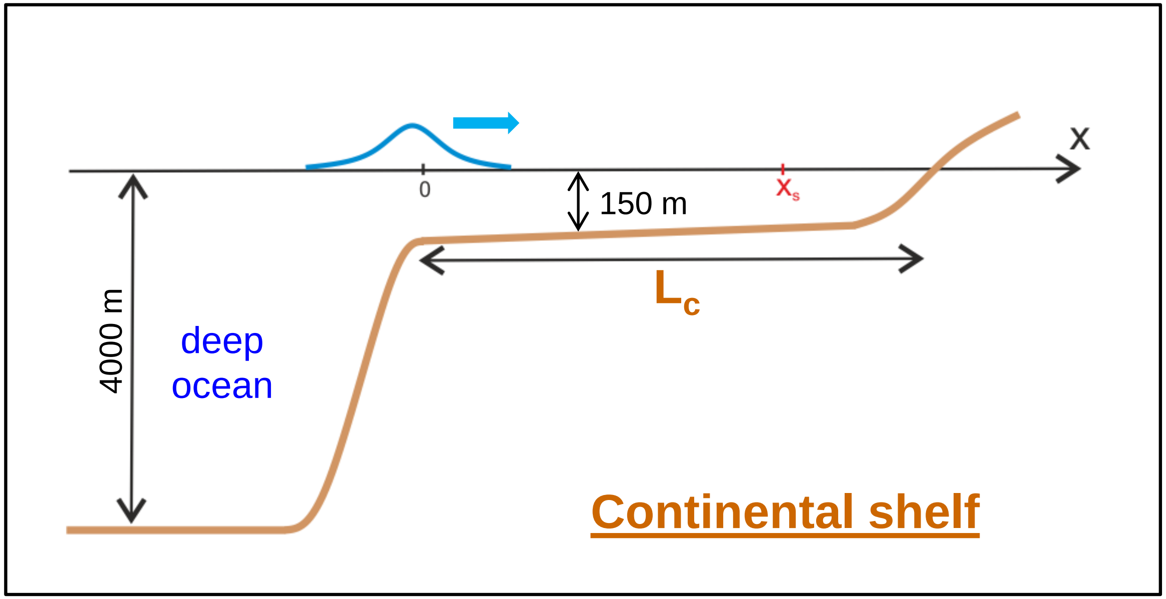


$$h(x_0, t=0) = D_0 + A_0(1 + \sin\frac{x_0}{L_{w0}})$$

$$x_s = \frac{2}{3}\frac{L_{w0}}{\epsilon_0}$$

see Madsen et al 2008

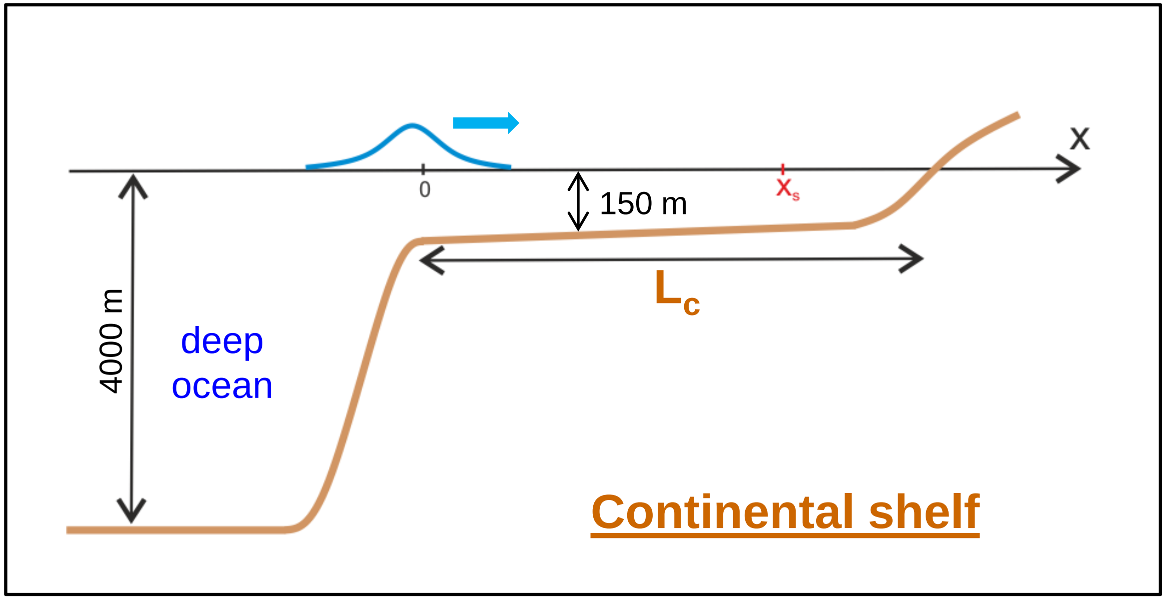
Basic conditions for bore formation



$$h(x_0, t=0) = D_0 + A_0(1 + \sin\frac{x_0}{L_{w0}})$$

$$x_s = \frac{2}{3}\frac{L_{w0}}{\epsilon_0} < L_c$$

Basic conditions for bore formation



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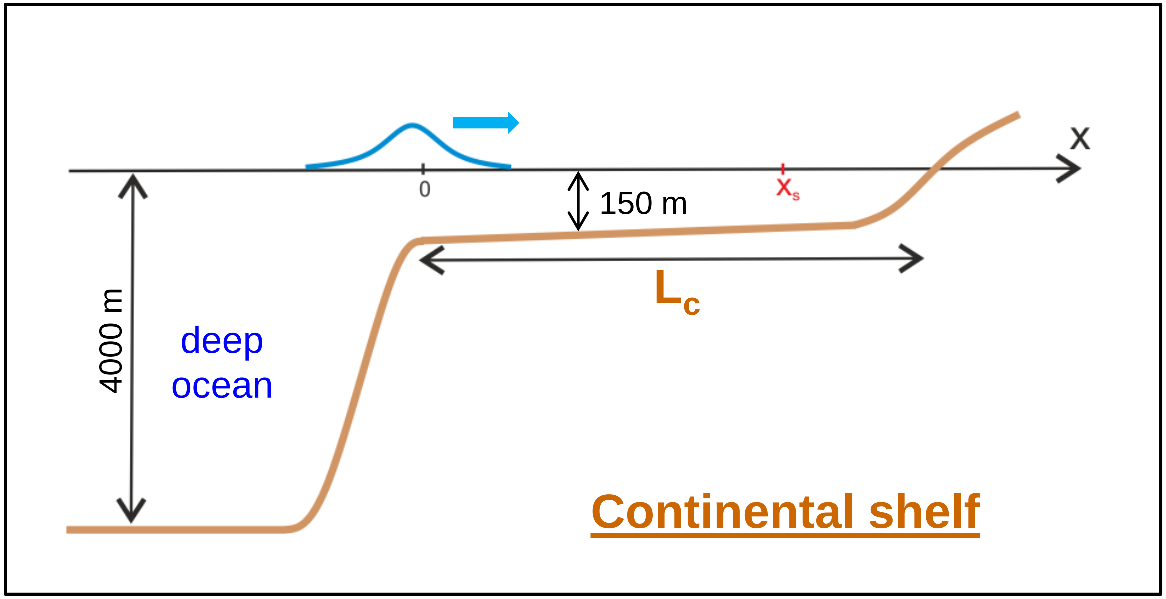
Basic conditions for bore formation



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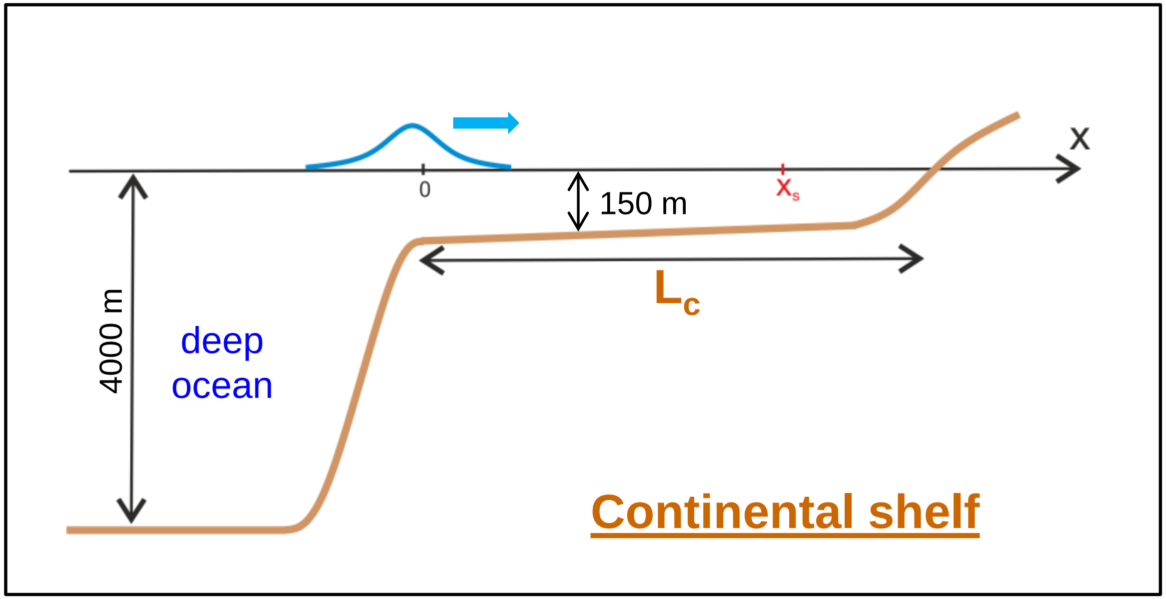
$$\delta_0 = \frac{L_{w0}}{L_c} < \frac{3}{2}\epsilon_0$$

Continental shelves $D_0 \approx 150$ m

- tsunamis: $A_0 \approx 2$ m, $T_0 \approx 25$ min $\Rightarrow$ $x_s = 460$ km
- tides: $T_0 \approx 744$ min $\Rightarrow$ $x_s >> L_c$ $\rightarrow$ no tidal bore

Basic conditions for bore formation



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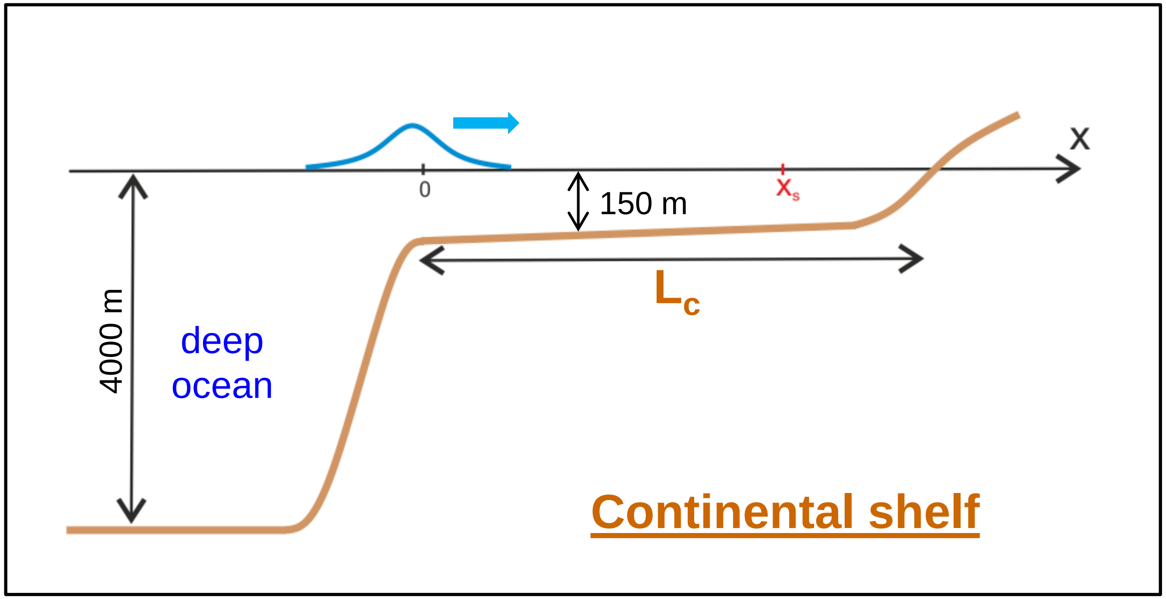
- tsunamis: bores may occur in large and shallow (few tens of m) coastal environments:

  marine coastal plains (e.g.: deltas, alluvial estuaries) or

  carbonate platforms (e.g.: coral reef systems)

Basic conditions for bore formation



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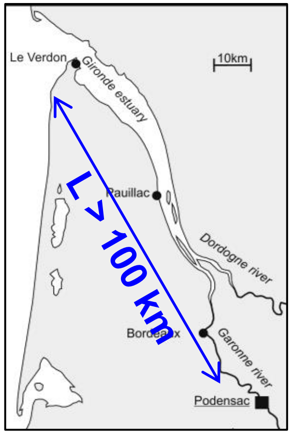
- tides: bores can occur in **long shallow alluvial estuaries**

  L ≈ 100 km D_0 ≈ 10 m



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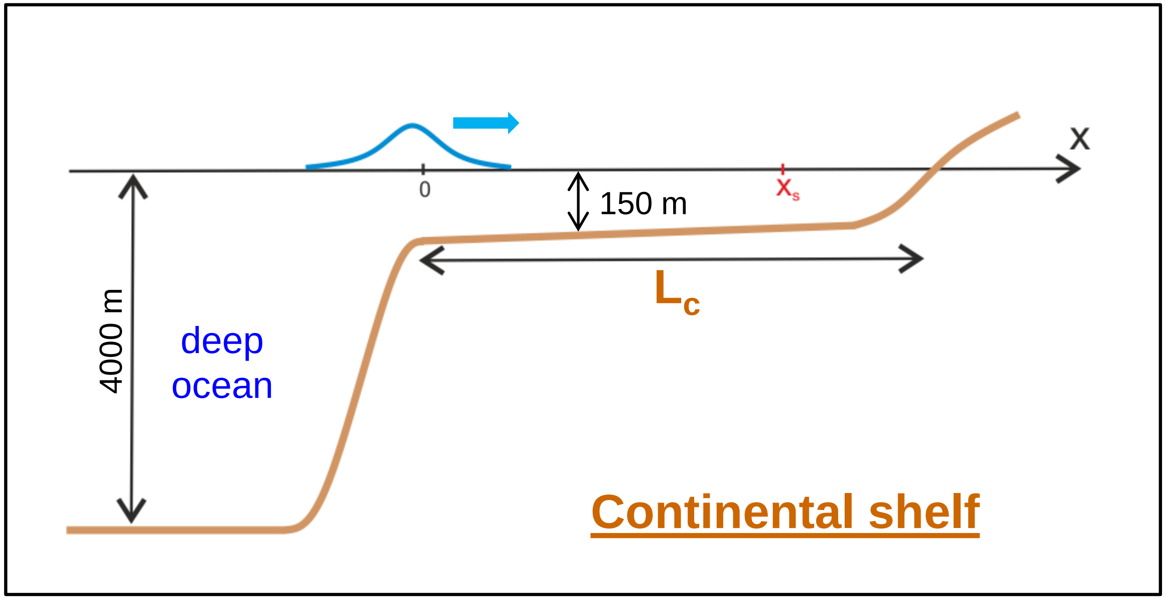

$$h(x_0, t=0) = D_0 + A_0(1 + \sin\frac{x_0}{L_{w0}})$$

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- tides: bores can occur in **long shallow alluvial estuaries**

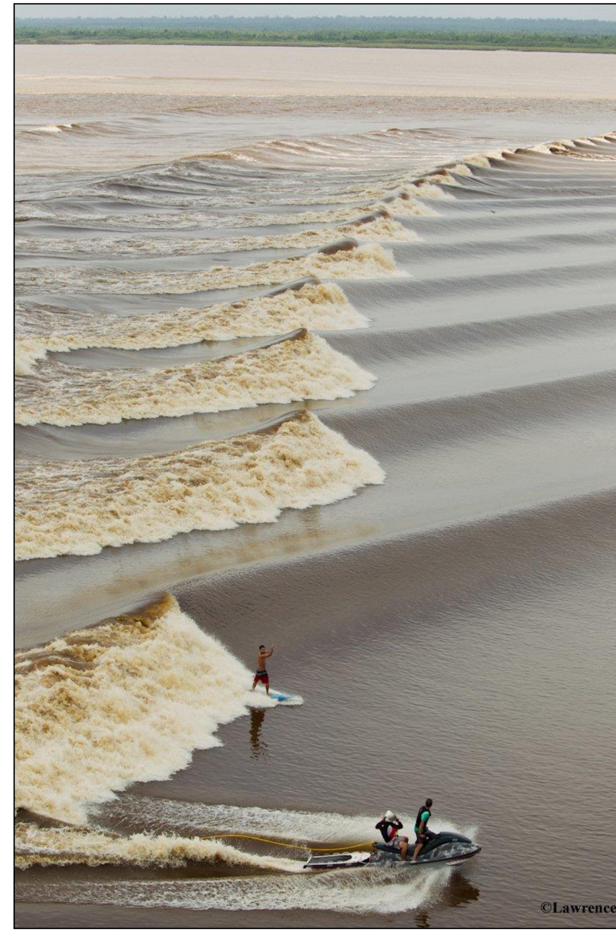
in such shallow environments friction can play a significant role

4. Dynamique des ressauts de marée et mascarets

Conditions for tidal bore formation | Worldwide tidal bores

Severn River - England

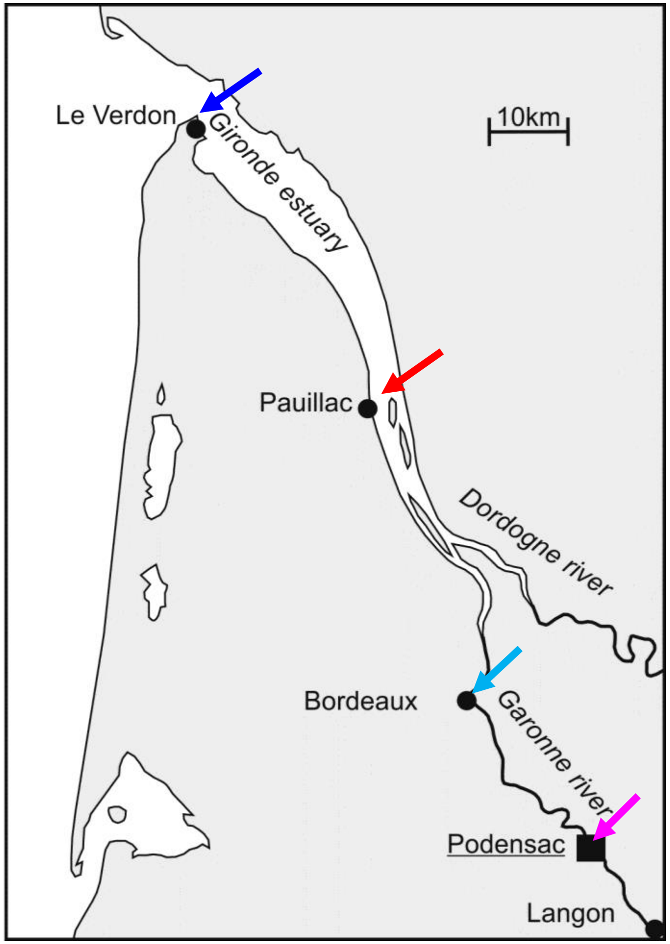
Amazon River – Brazil (Pororoca)

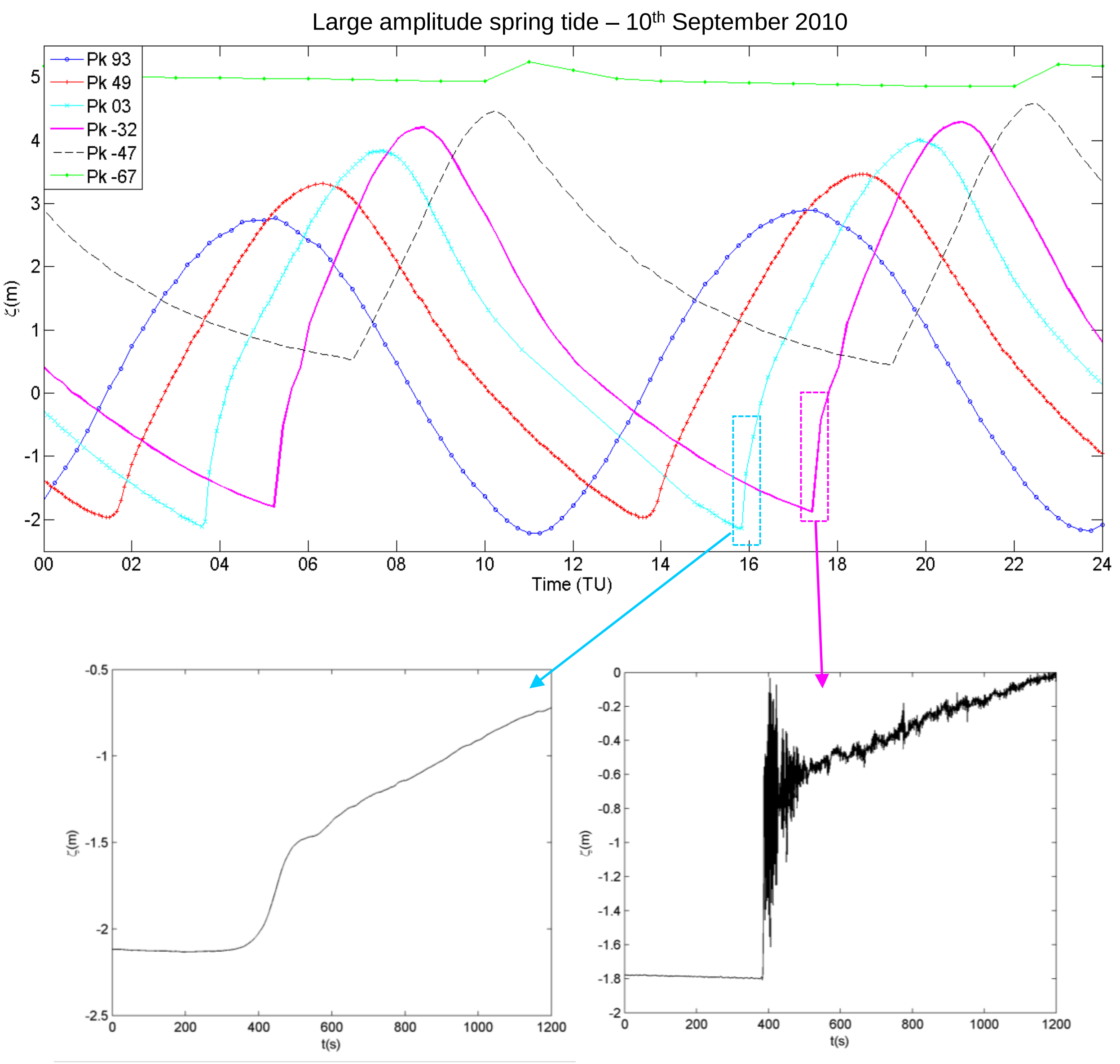


Kampar River – Sumatra (Bono)

Qiantang River – China

Conditions for tidal bore formation | Worldwide tidal bores

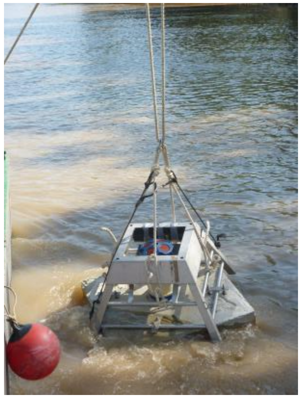




Bonneton et al. JGR 2015

Gironde/Garonne/Dordogne estuary – France

3 field campaigns : a unique long-term high-frequency database



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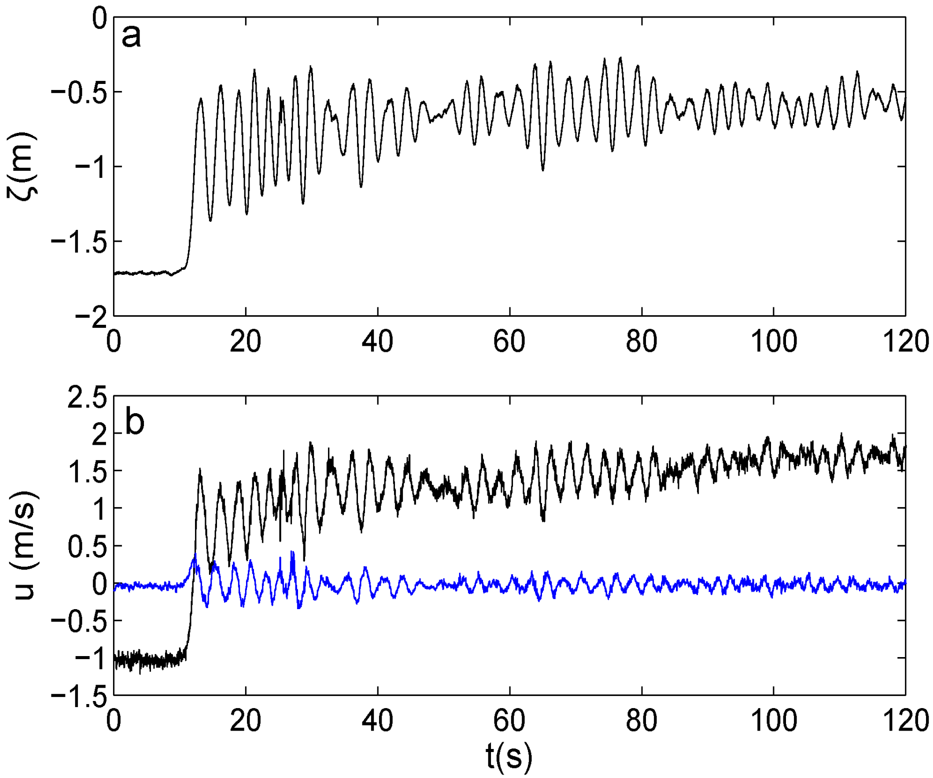


Bonneton et al. JGR 2015

Conditions for tidal bore formation | Physical criteria

- **Large tidal range** ($Tr_0=2A_0$) $\rightarrow$ Chanson (2012) : ~~$Tr_0 > 4.5\text{-}6$ m~~
- **Small water depth**
- **Large-scale funnel-shaped estuaries**

} coastal plain alluvial estuaries

$\Rightarrow$ **Scaling analysis**

- ❑ **Large tidal range**
- ❑ **Small water depth**
- ❑ **Large-scale funnel-shaped estuaries**

} coastal plain alluvial estuaries

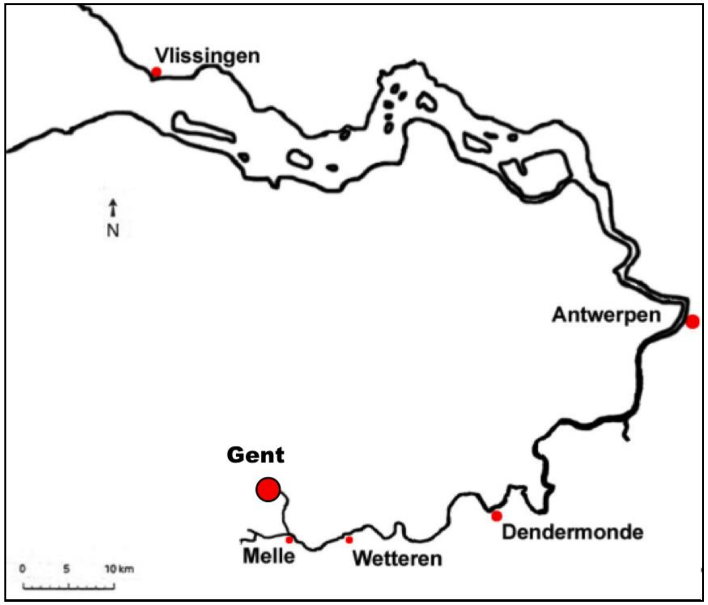
⇒ Scaling analysis

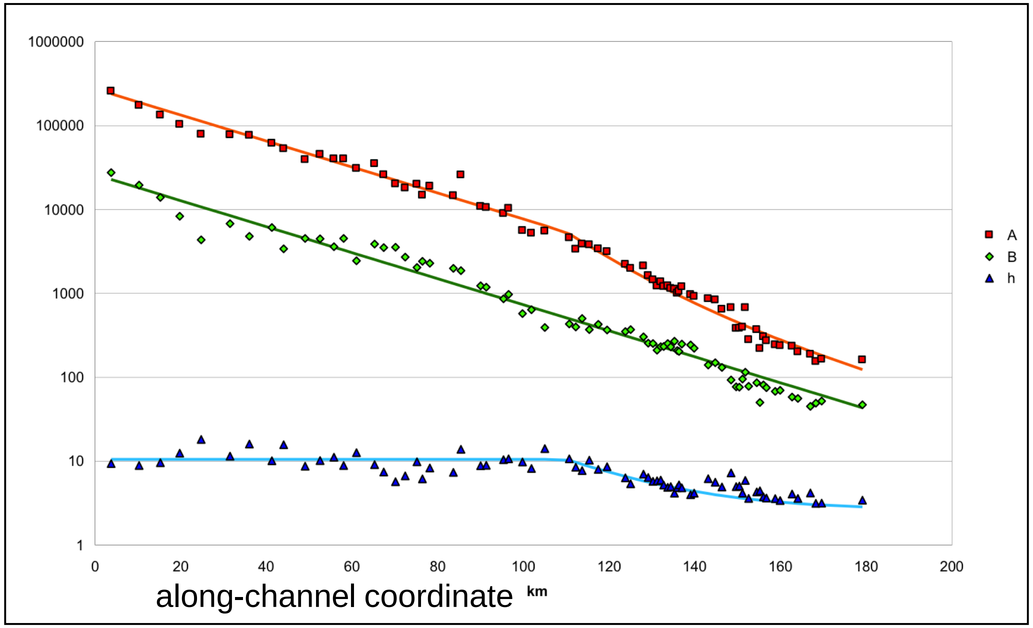
Identify the characteristic scales of the problem:

- morphology of alluvial estuaries
- tidal waves

Tide-dominated alluvial estuaries show many morphological similarities all over the world

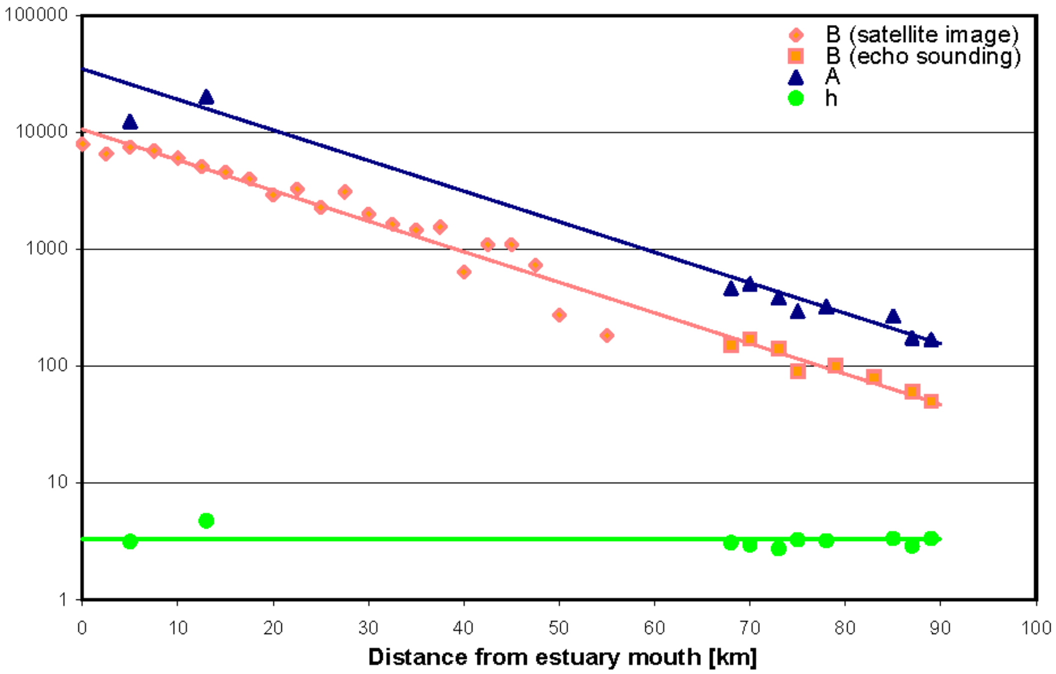
Scheldt estuary





Savenije 2012

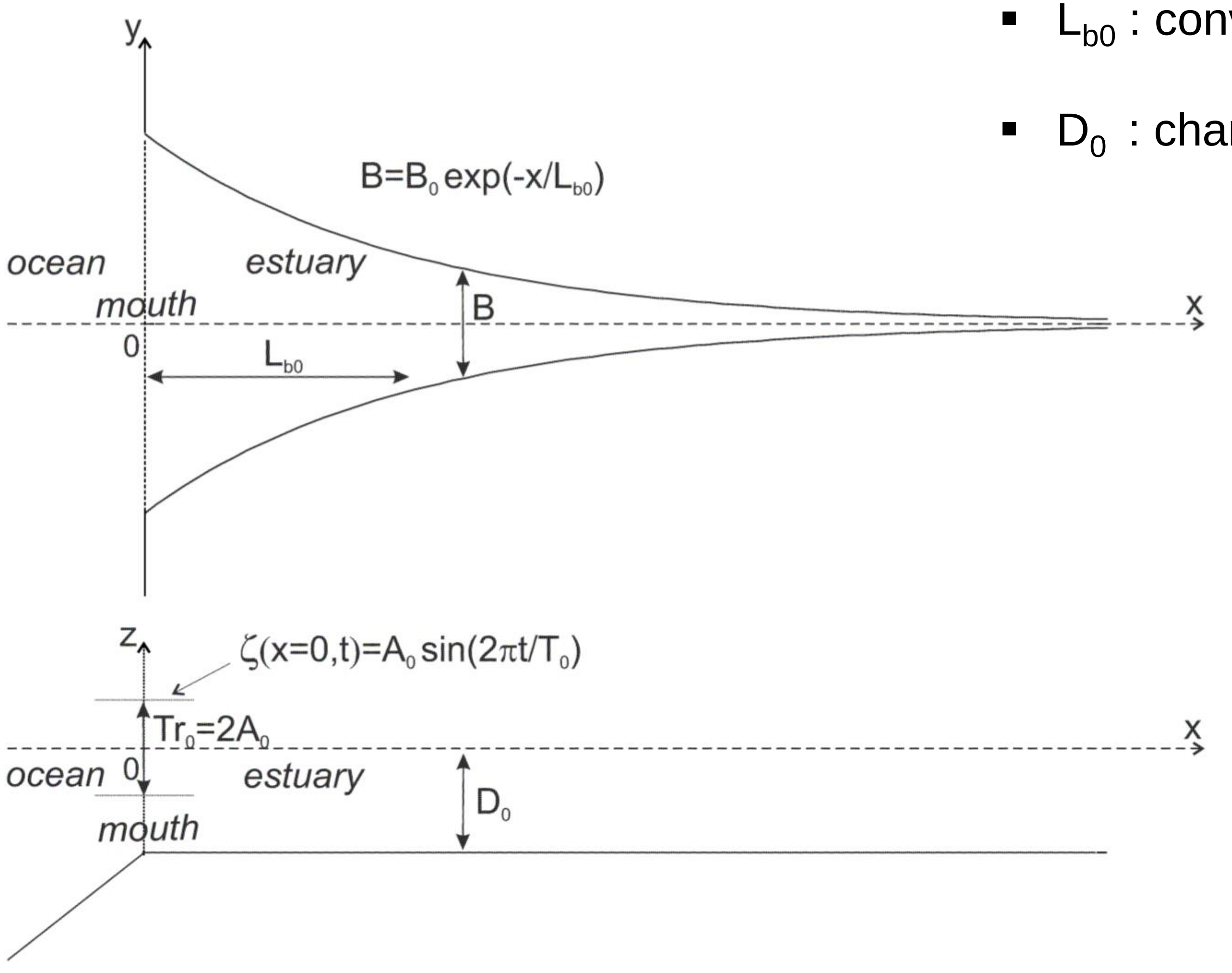
Pungue estuary



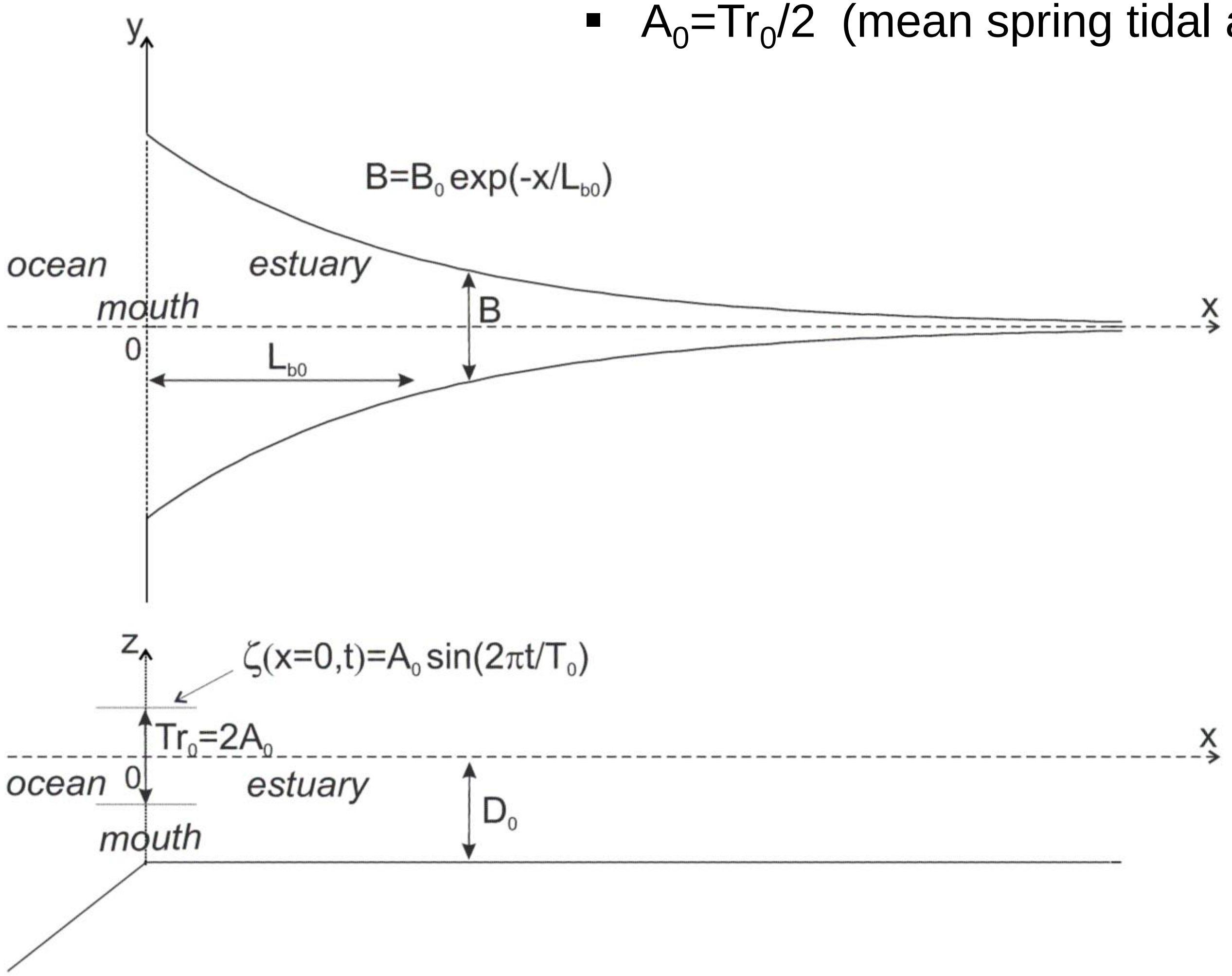
Graas et al. 2008

$$B = B_0 e^{-x/L_{B0}}$$

- L_{b0} : convergence length
- D_0 : characteristic water depth

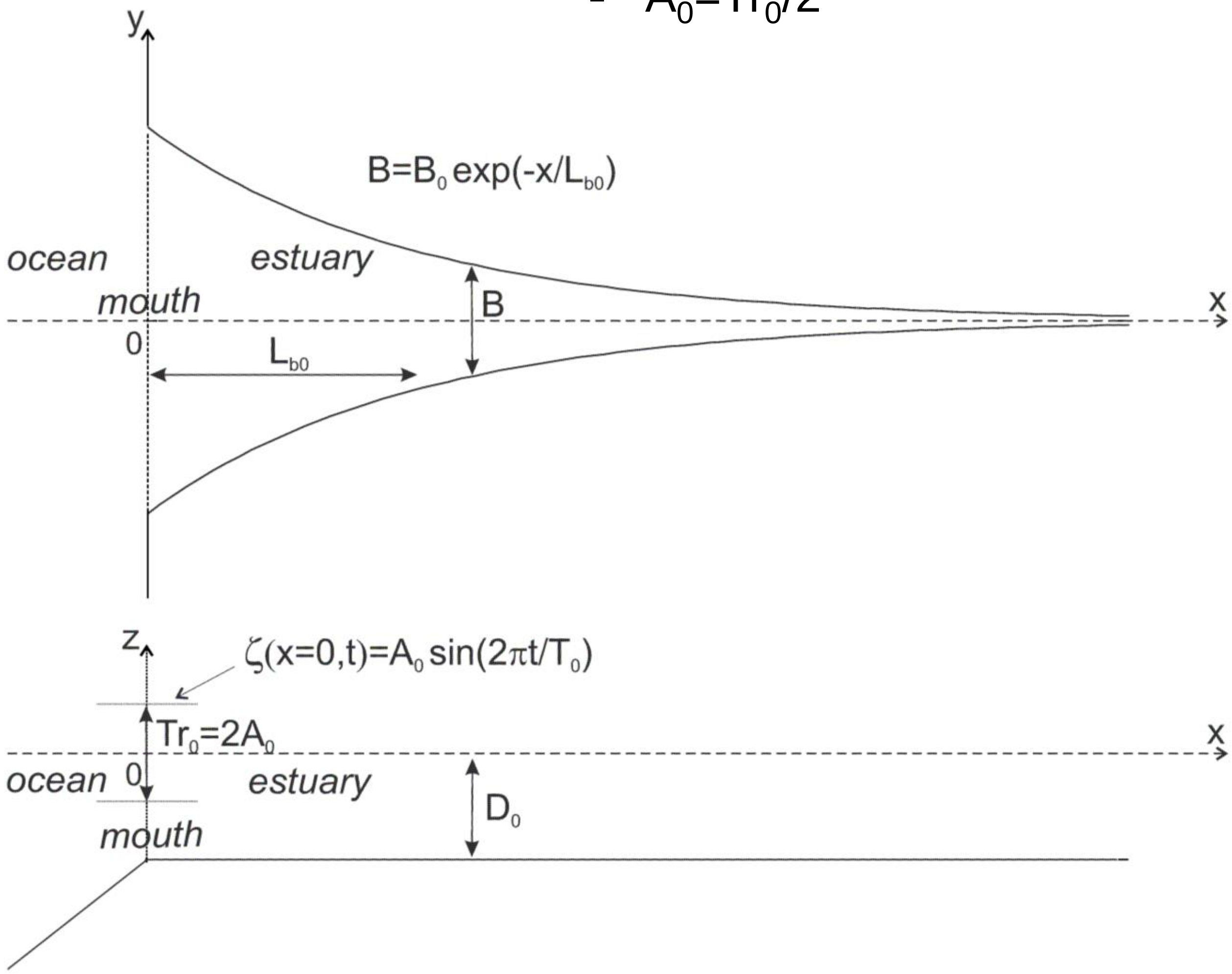


- L_{b0}
- D_0
- T_0=12.4 h → $L_{w0} = \sqrt{gD_0}/\omega_0$
- $A_0=Tr_0/2$ (mean spring tidal amplitude)



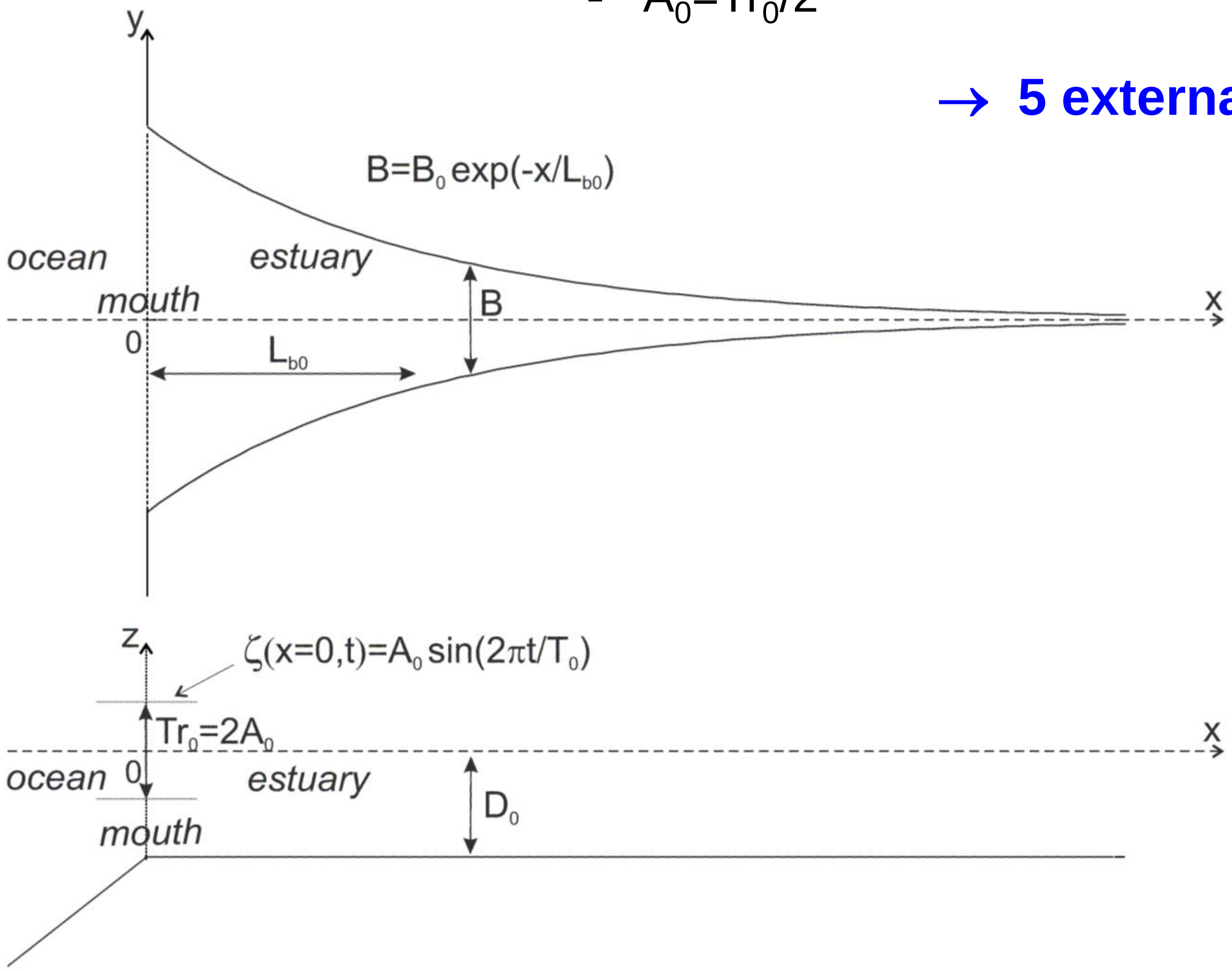
- L_{b0}
- D_0
- T_0=12.4 h
- $A_0=Tr_0/2$

- Cf_0 : friction coefficient
- Q_0 : freshwater discharge



- L_{b0}
- D_0
- T_0=12.4 h
- A_0=Tr_0/2
- Cf_0 : friction coefficient

→ 5 external variables



$$\frac{\partial \zeta}{\partial t} + u\frac{\partial \zeta}{\partial x} + D\frac{\partial u}{\partial x} + \frac{u}{B}\frac{\partial \mathcal{A}}{\partial x})_{z=\zeta} = 0$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + g\frac{\partial \zeta}{\partial x} + C_{f0}\frac{|u|u}{D} = 0$$

$$B = B_0 \exp(-\frac{x}{L_{b0}})$$

$$\frac{1}{B}\frac{\partial \mathcal{A}}{\partial x})_{z=\zeta} = -\frac{D}{L_{b0}}$$

$$t' = \frac{t}{T_0/2\pi}, \quad D' = \frac{D}{D_0}, \quad \zeta' = \frac{\zeta}{A_0}, \quad x' = \frac{x}{L_0}, \quad u' = \frac{u}{U_0} .$$

$$\frac{\partial \zeta}{\partial t} + \frac{K}{\mathcal{L}}\left(\epsilon_0 u \frac{\partial \zeta}{\partial x} + D\frac{\partial u}{\partial x}\right) - KuD = 0$$

$$\frac{\partial u}{\partial t} + \frac{K}{\mathcal{L}}\epsilon_0 u \frac{\partial u}{\partial x} + \frac{1}{K\mathcal{L}}\delta_0^2 \frac{\partial \zeta}{\partial x} + \underbrace{K\frac{\epsilon_0 \phi_0}{\delta_0}}_{\mathcal{D}_i}\frac{|u|u}{D} = 0\,.$$

$$\epsilon_0 = \frac{A_0}{D_0} \qquad \delta_0 = \frac{L_{w0}}{L_{b0}} \qquad \phi_0 = \frac{C_{f0}L_{w0}}{D_0}$$

$$L_{w0} = (gD_0)^{1/2}\omega_0^{-1}$$

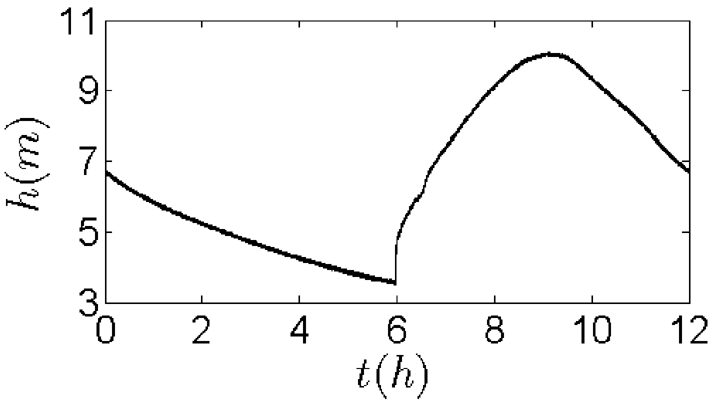
$$K = \frac{U_0}{L_{b0}A_0D_0^{-1}\omega_0} \qquad \mathcal{L} = \frac{L_0}{L_{b0}}$$

$$\frac{\partial \zeta}{\partial t} + \frac{K}{\mathcal{L}}\left(\epsilon_0 u \frac{\partial \zeta}{\partial x} + D\frac{\partial u}{\partial x}\right) - KuD = 0$$
$$\frac{\partial u}{\partial t} + \frac{K}{\mathcal{L}}\epsilon_0 u \frac{\partial u}{\partial x} + \frac{1}{K\mathcal{L}}\delta_0^2 \frac{\partial \zeta}{\partial x} + \underbrace{K\frac{\epsilon_0 \phi_0}{\delta_0}}_{\mathcal{D}_i}\frac{|u|u}{D} = 0\,.$$

K $\approx$ 1, for tidal bore estuaries

$$D_i = \frac{C_{f0} L_{b0} A_0}{{D_0}^2} > 1.5$$

Bonneton et al., JGR 2015



→ necessary condition for tidal bore formation by not a sufficient one

$$\frac{\partial \zeta}{\partial t} + \frac{K}{\mathcal{L}}\left(\epsilon_0 u \frac{\partial \zeta}{\partial x} + D\frac{\partial u}{\partial x}\right) - KuD = 0$$

$$\frac{\partial u}{\partial t} + \frac{K}{\mathcal{L}}\epsilon_0 u \frac{\partial u}{\partial x} + \frac{1}{K\mathcal{L}}\delta_0^2 \frac{\partial \zeta}{\partial x} + \underbrace{K\frac{\epsilon_0 \phi_0}{\delta_0}}_{\mathcal{D}_i} \frac{|u|u}{D} = 0\,.$$

→ **explore the 3D dimensionless external parameter space: (ε_0 , δ_0 , ϕ_0)**

- Field data: 21 convergent alluvial estuaries

  Bonneton, P., Filippini, A.G., Arpaia, L., Bonneton, N. and Ricchiuto, M 2016.
  Conditions for tidal bore formation in convergent alluvial estuaries. *ECSS,* **172**, 121-127

- Numerical simulations: 225 runs of a shallow water model

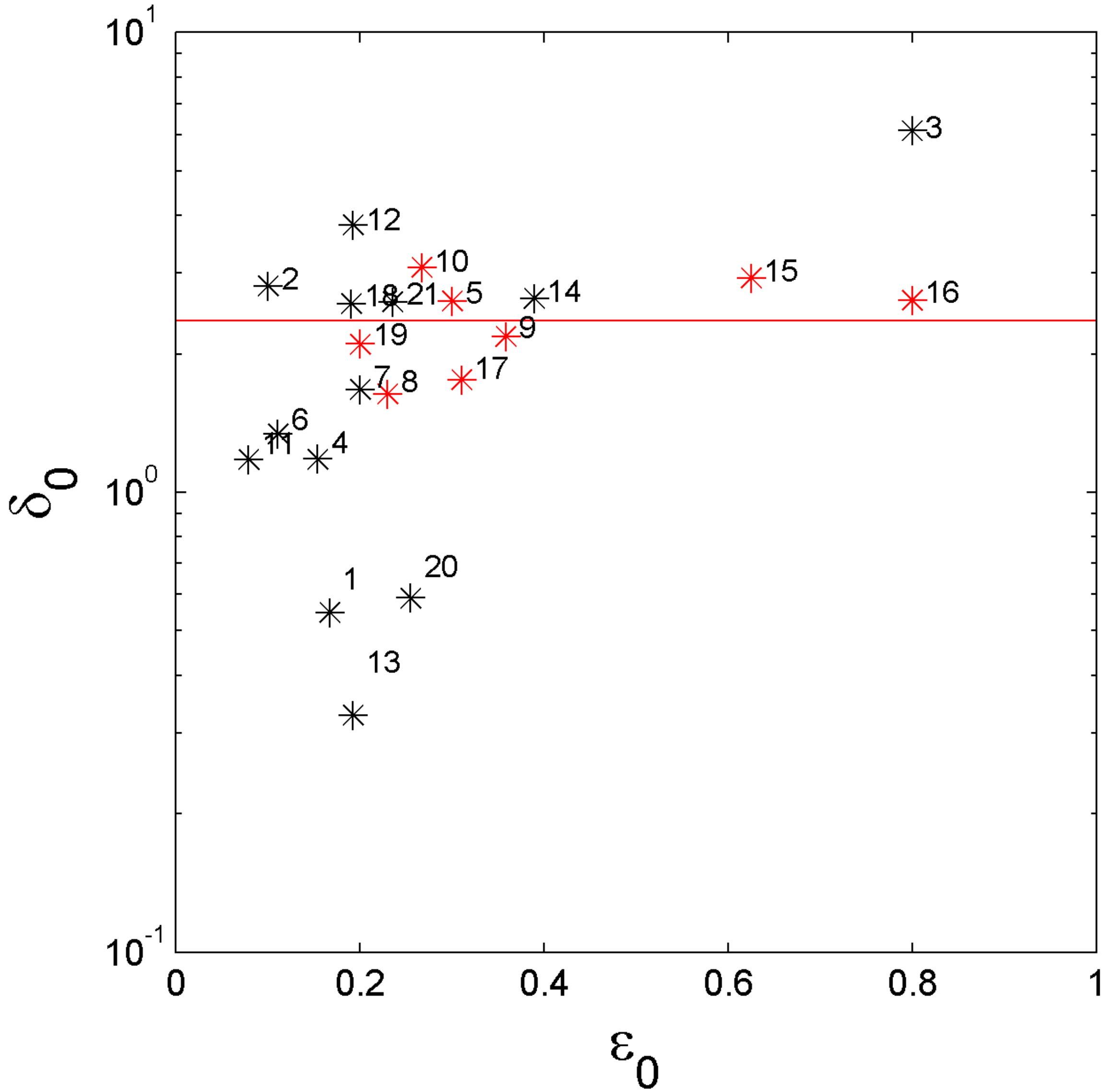
  Filippini, A.G., Arpaia, L., Bonneton, P., and Ricchiuto, M. 2017.
  Modelling analysis of tidal bore formation in convergent estuaries. in revision

1	Chao Phya	Thailand
2	Columbia	USA
3	Conwy	UK
4	Corantijn	USA
5	Daly	Australia
6	Delaware	USA
7	Elbe	Germany
8	Gironde	France
9	Hooghly	India
10	Humber	UK
11	Limpopo	Mozambique
12	Loire	France
13	Mae Klong	Thailand
14	Maputo	Mozambique
15	Ord	Australia
16	Pungue	Mozambique
17	Qiantang	China
18	Scheldt	Netherlands
19	Severn	UK
20	Tha Chin	Thailand
21	Thames	UK

- 21 convergent alluvial estuaries
- 9 tidal bore estuaries

D_0, L_{b0}, $A_0 = Tr_0/2$, Cf_0

Conditions for tidal bore formation — Field data



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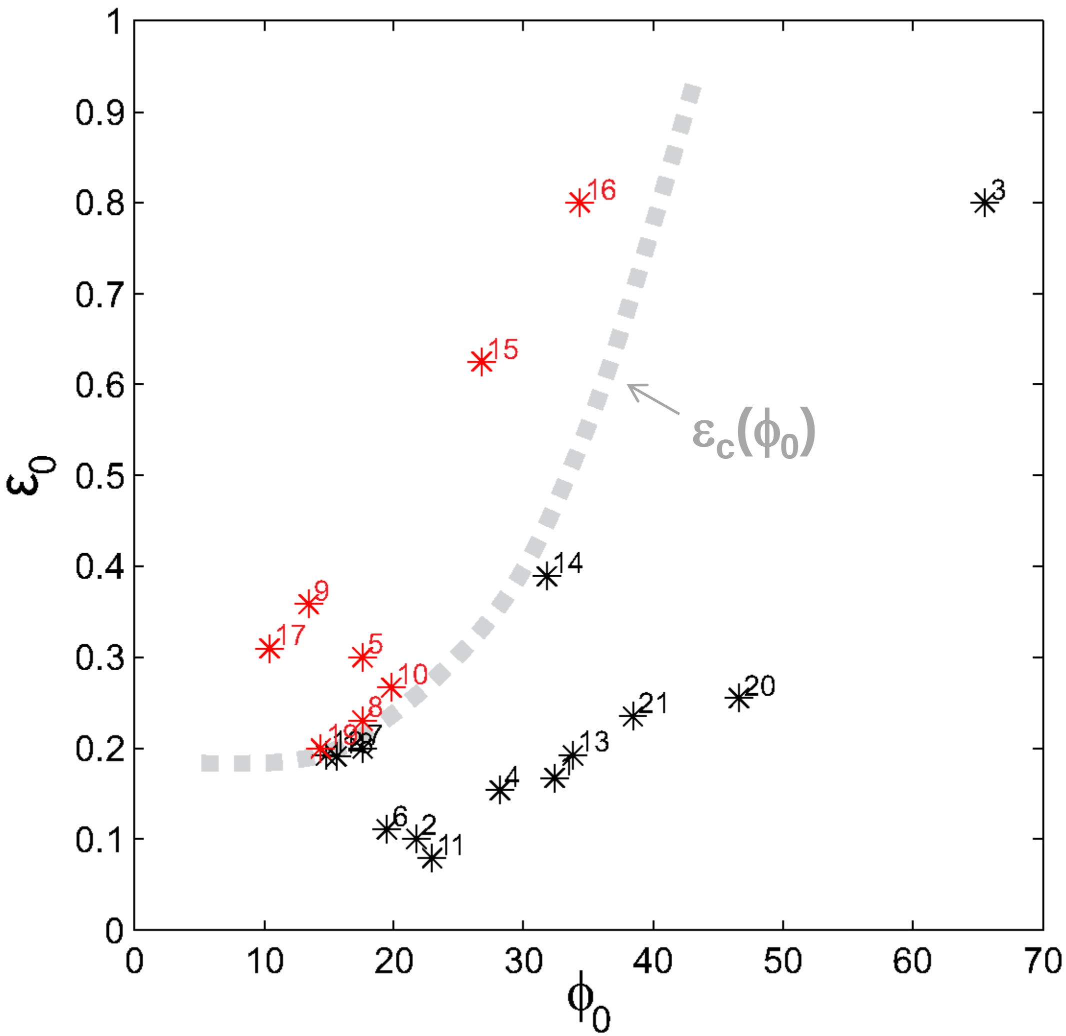
Tidal bore estuaries: $\delta_0 \approx 2.4 \rightarrow$ 2D parameter space (ε_0 , ϕ_0)

Conditions for tidal bore formation | Field data



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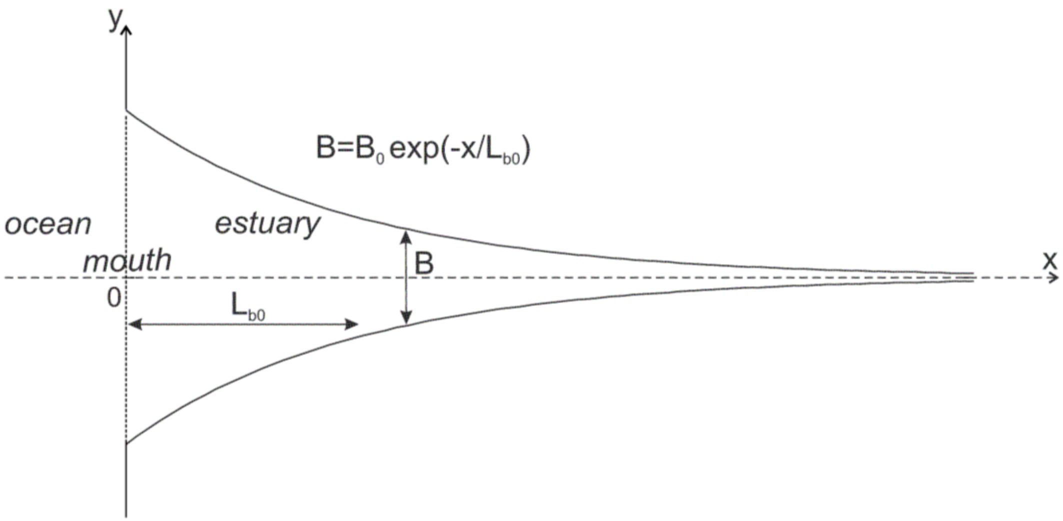
19	Severn	UK
20	Tha Chin	Thailand
21	Thames	UK

Tidal bores occur when $\varepsilon_0 > \varepsilon_c(\phi_0)$

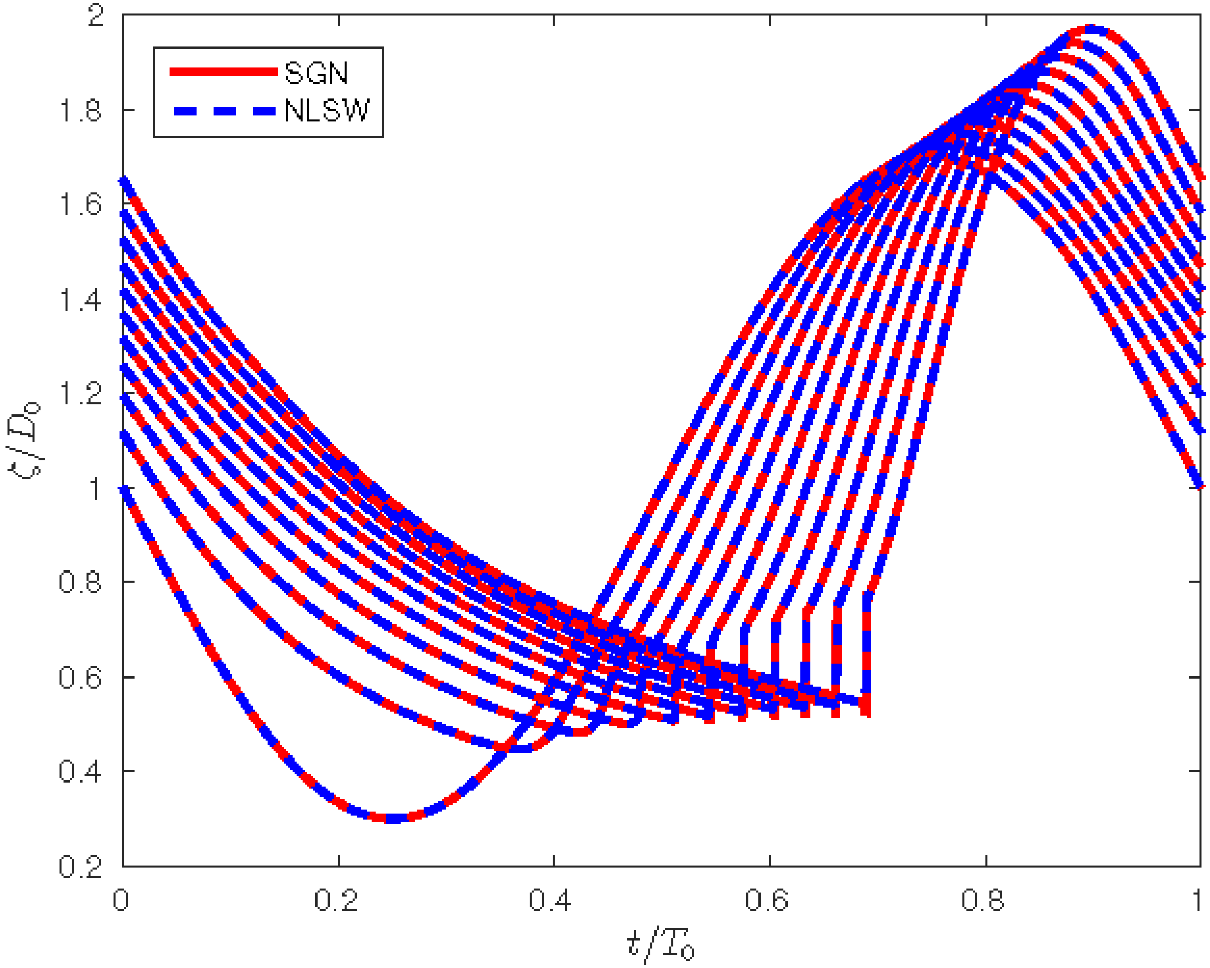
Numerical investigation of the 2D parameter space (ε_0 , ϕ_0)

→ **225 runs** with $\delta_0 = 2$

Filippini, A.G., Arpaia, L., Bonneton, P., and Ricchiuto, M. 2017

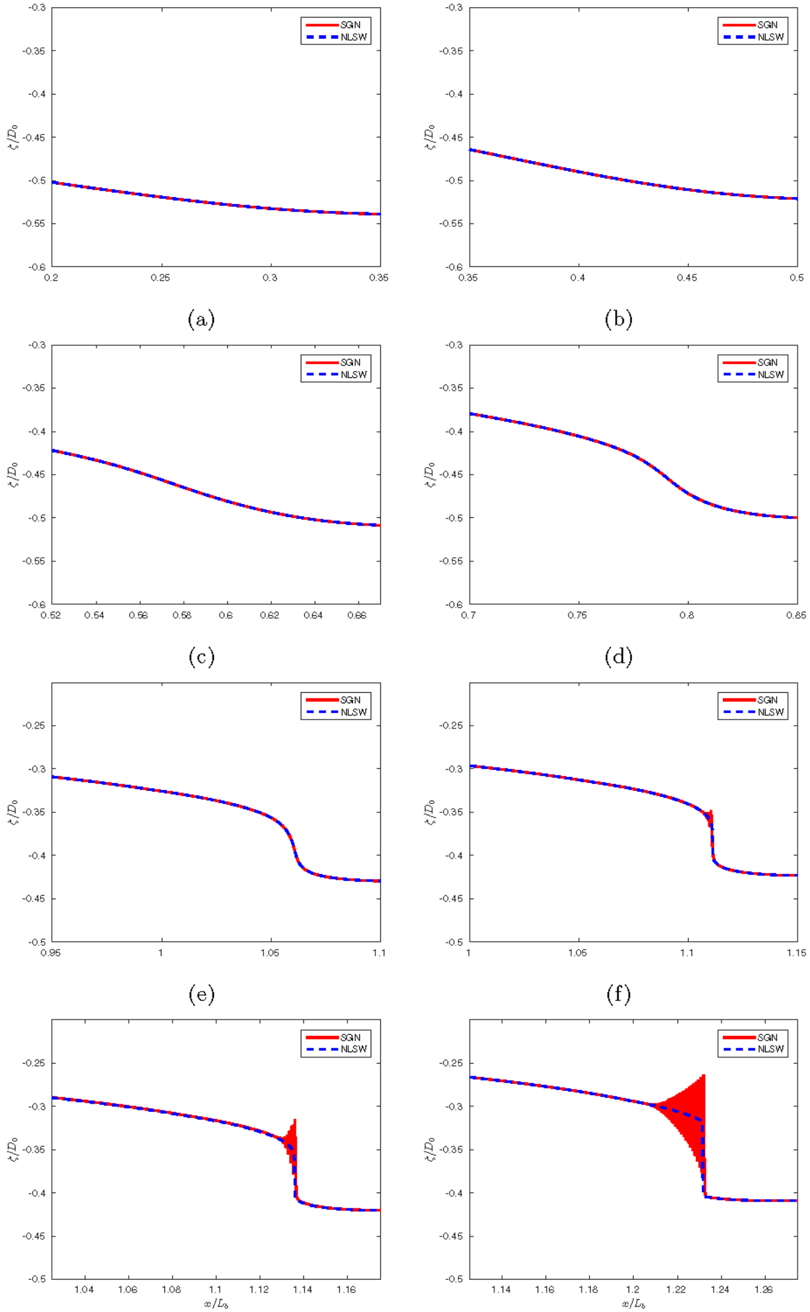


SGN / SV

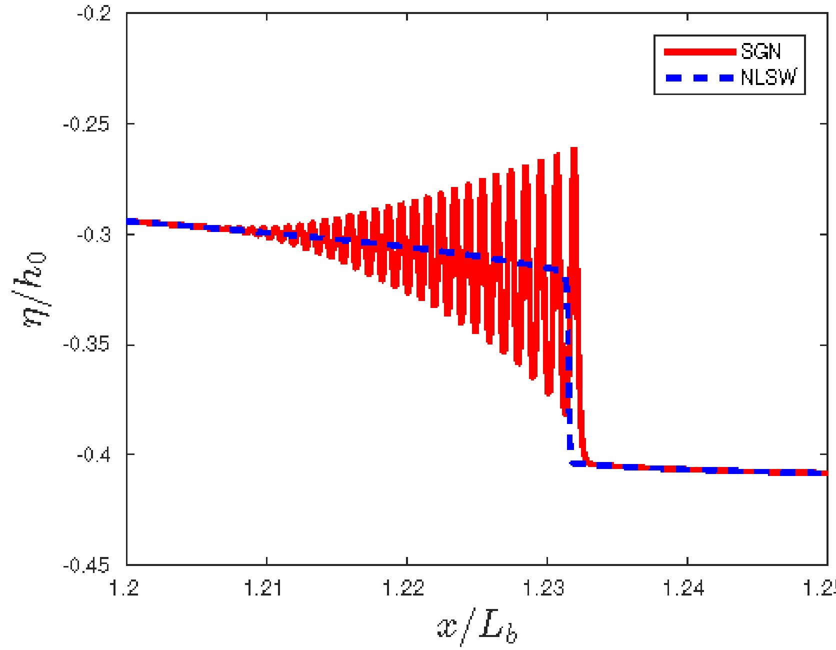


Filippini et al. 2017

Conditions for tidal bore formation | Numerical simulations



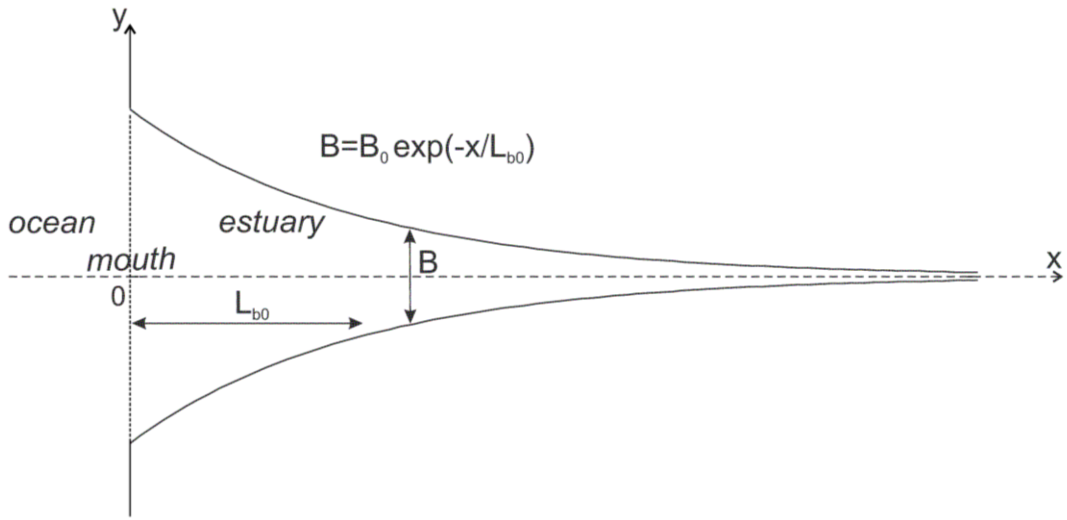
SGN / SV



Filippini et al. 2017

Numerical investigation of the 2D parameter space (ε_0 , ϕ_0)

→ **225 runs** with $\delta_0 = 2$

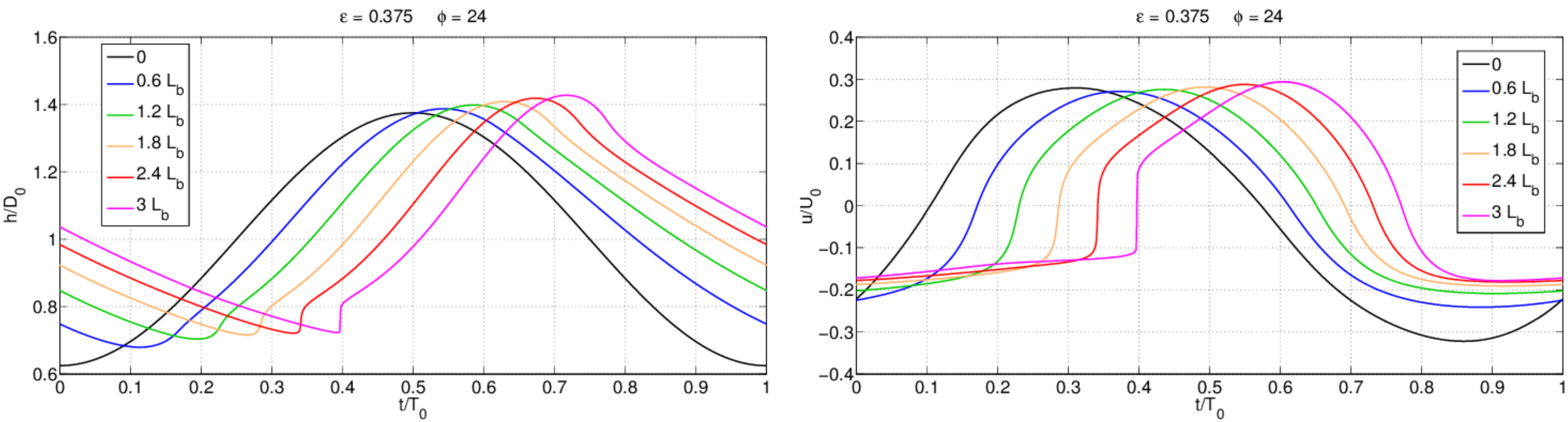


2D nonlinear shallow water model developed by *Ricchiuto, JCP 2015*

- shock capturing residual distribution scheme
- 2^{nd} order in space and time
- unstructured grids suitable for real estuarine applications

Filippini et al. 2017

one example on the 225 runs



$$S_{max} = max\left(\frac{\partial \zeta}{\partial x}\right)$$

Filippini et al. 2017

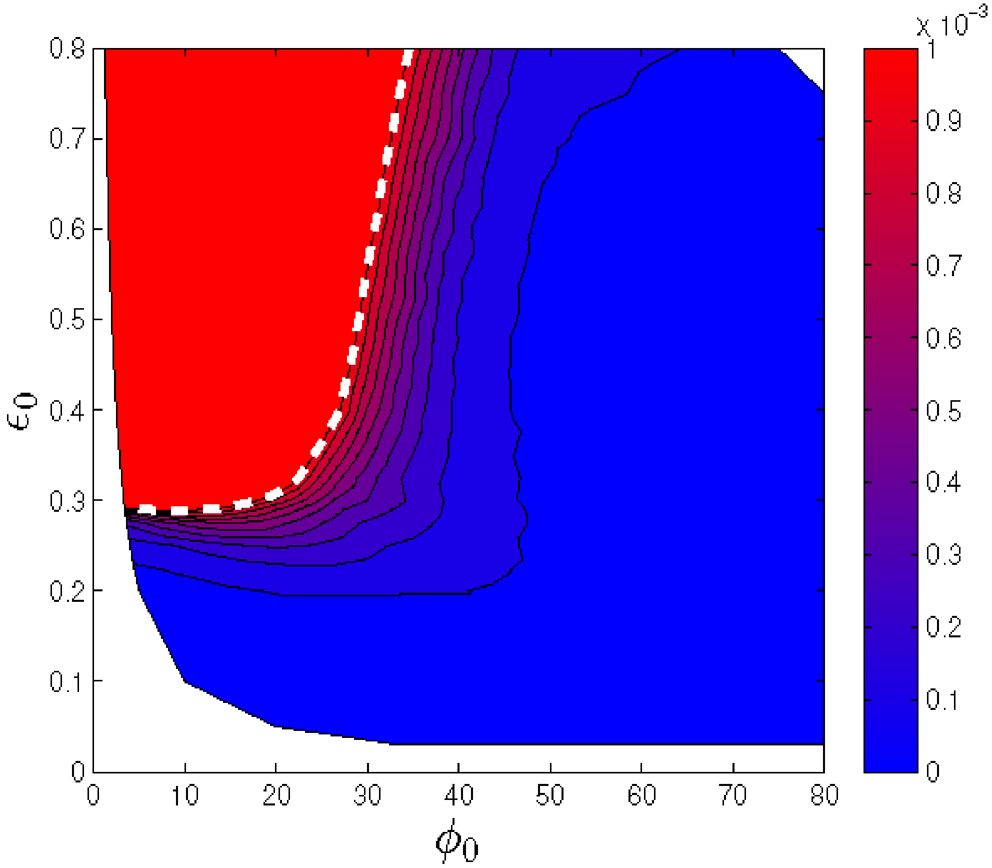
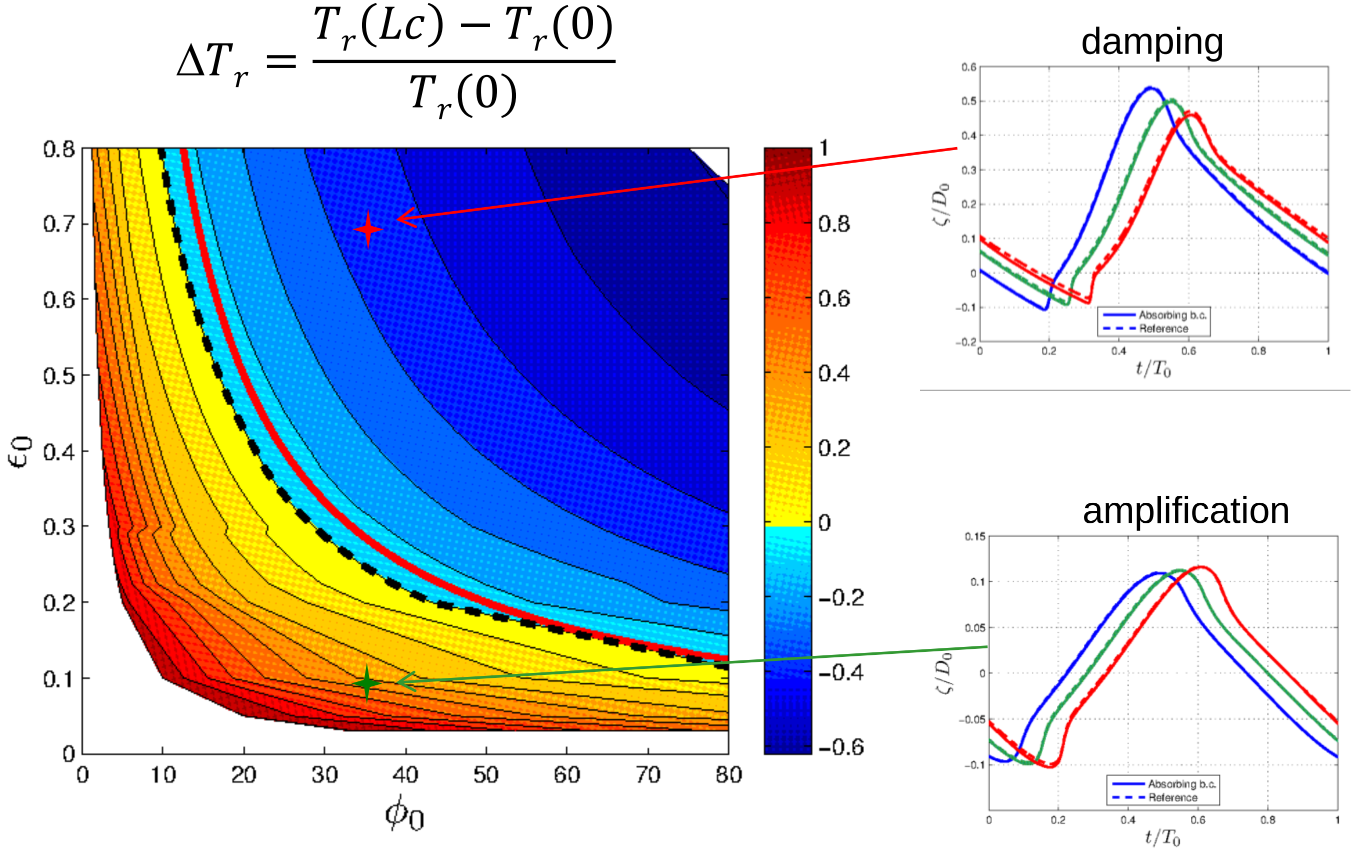


Figure 2: Isocurves of the quantity S_{max} in the plane of the parameters (ϕ_0, ϵ_0), the white dashed line represents the $\epsilon_c(\phi_0)$ curve, namely the limit for tidal bore appearance following the criterion $S_{max} \geq 10^{-3}$.

Filippini et al. 2017

rate of change of the tidal range

$$\Delta T_r = \frac{T_r(Lc) - T_r(0)}{T_r(0)}$$



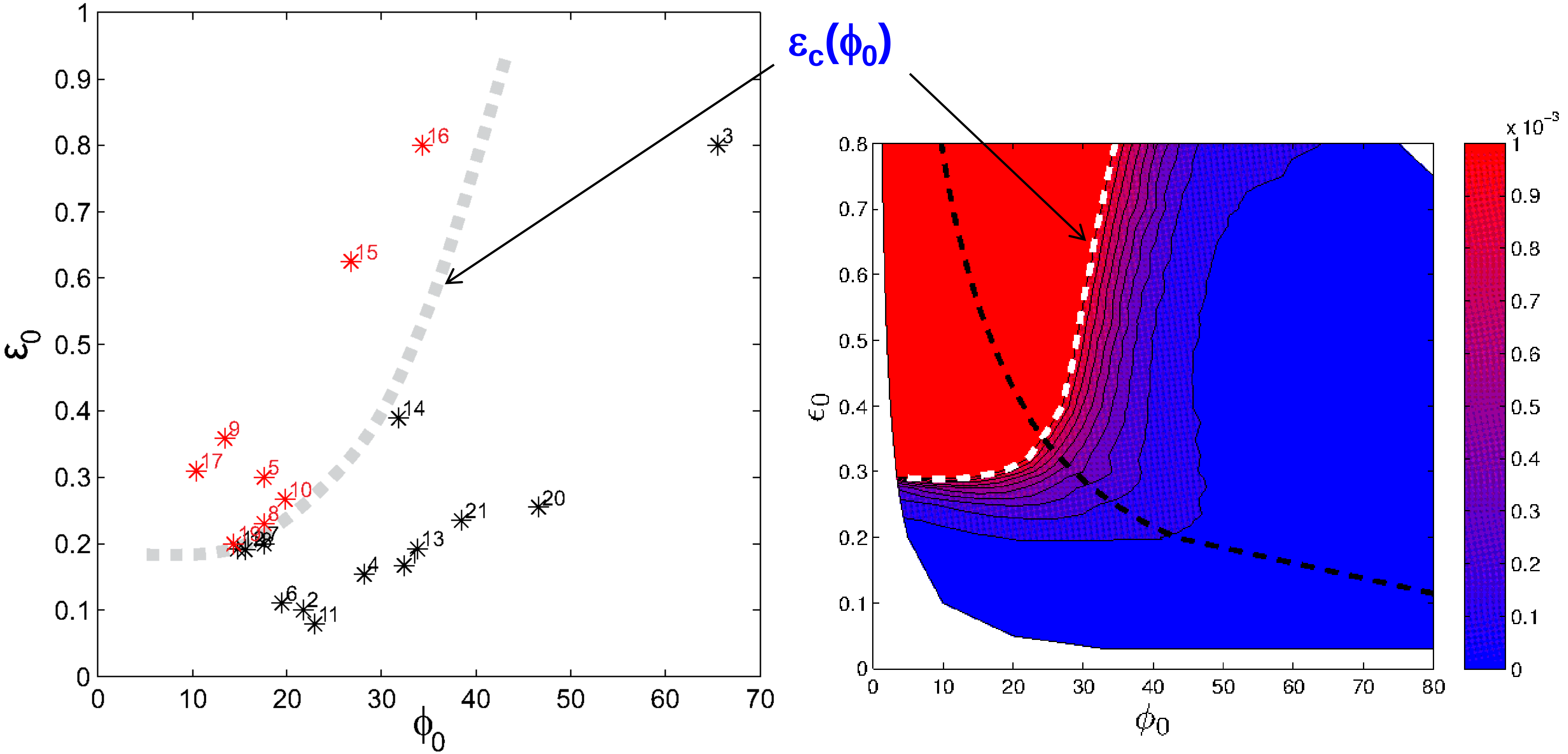
$$\frac{\partial u}{\partial t} + \frac{1}{\mathcal{L}}\epsilon_0 u \frac{\partial u}{\partial x} + \frac{1}{\mathcal{L}}\delta_0^2 \frac{\partial \zeta}{\partial x} + \underbrace{\frac{\epsilon_0 \phi_0}{\delta_0}}_{\mathcal{D}_i} \frac{|u|u}{D} = 0$$

Theoretical zero-amplification curve:

$\varepsilon_0 \phi_0 = \delta_0(\delta_0^2 + 1)$ *(Savenije et al. 2008)*

Filippini et al. 2017

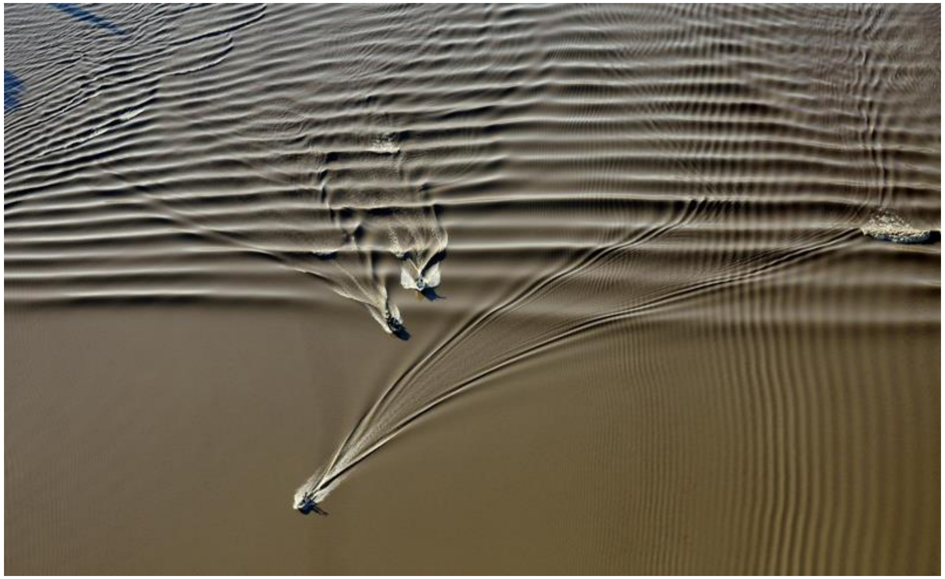
Conditions for tidal bore formation | Estuary classification



Tidal bores occur when $\varepsilon_0 > \varepsilon_c(\phi_0)$

Ondes longues et chocs dispersifs

tsunami



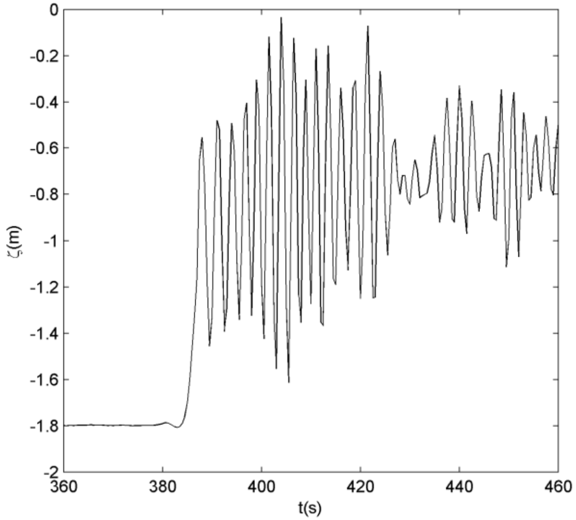
ressaut de marée (mascaret)

Conclusion

Large scale phenomenon
→ tidal wave

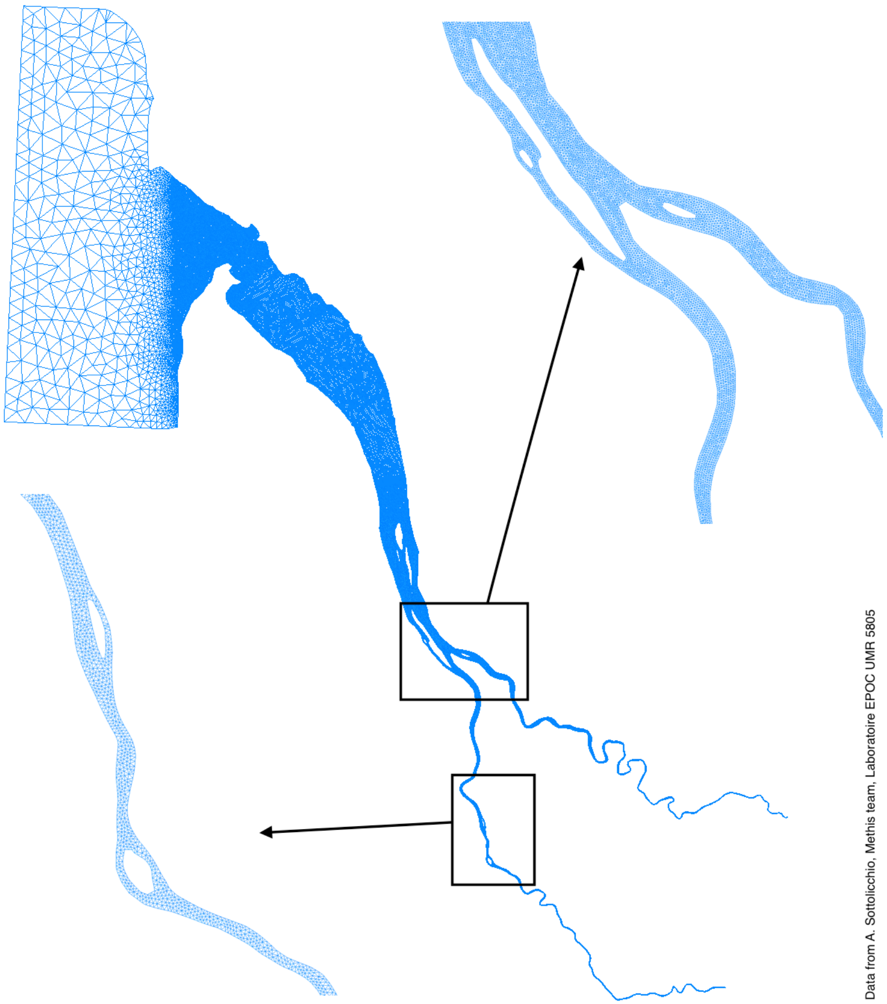
- $L_{TW} \approx 100$ km
- $T_{TW} \approx 12.4$ h

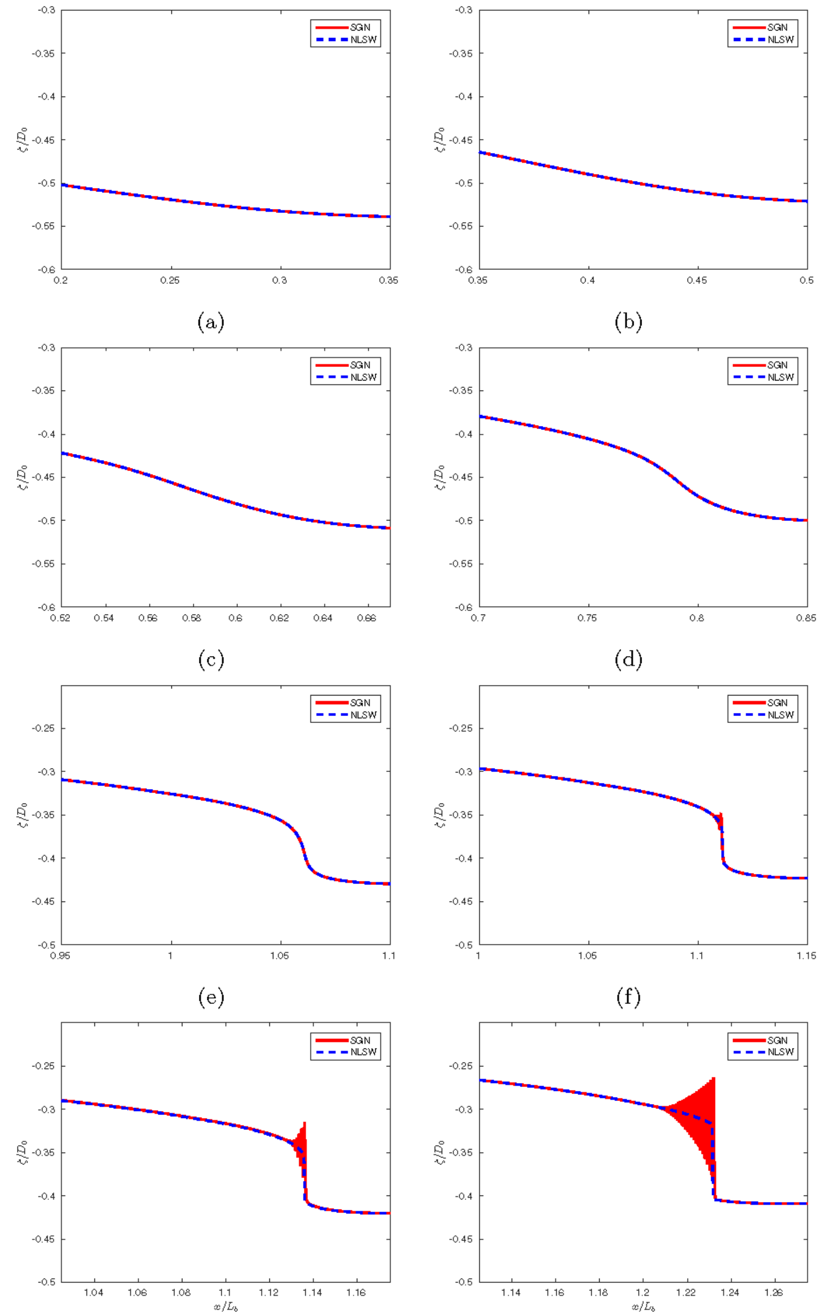
Small scale wave phenomenon
→ tidal bore



- $L_{TB} \approx 10$ m
- $T_{TB} \approx 1$ s

Conclusion


Data from A. Sottolicchio, Methis team, Laboratoire EPOC UMR 5805


SGN
NLSW
ζ/D_0
x/L_0
(a)
(b)
(c)
(d)
(e)
(f)

Thank you for your attention